MONASH University

MONASH BUSINESS SCHOOL

Department of Economics
ISSN number 1441-5429

Financial Fraud and Investor Awareness

Discussion Paper no. 2021-06

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Keywords: financial fraud, investor naivety, unawareness, shrouding

JEL Classification: D14, D83, G11

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Financial Fraud and Investor Awareness*

Zhengqing Gui† Yangguang Huang‡ Xiaojian Zhao§

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*We thank Sarah Auster, Yan Chen, Kim-Sau Chung, Yuk-Fai Fong, Kohei Kawamura, Martin Peitz, Jean Paul Rabanal, Olga A. Rud, Burkhard C. Schipper, Heiner Schumacher, Ernst-Ludwig von Thadden, and Jidong Zhou for their stimulating and helpful suggestions. We also thank seminar and conference participants at the Chinese University of Hong Kong (Shenzhen), East China Normal University, Hong Kong Baptist University, Hong Kong University of Science and Technology, Monash University, Nankai University, Osaka University, University of Melbourne, CMES 2018, EEA-ESEM 2018, EARIE 2018, ASSA 2019, Tsinghua BEAT 2019, and Zoom Mini-Workshops on Unawareness 2021 for valuable comments. Huang acknowledges the support from HKUST Institute for Emerging Market Studies (Grant No. IEMS19BM01). Zhao acknowledges the support from Monash Economics Department (B03005-1758334).

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1 Introduction

Financial fraud refers to firms taking deceptive actions to exploit investors, such as Ponzi schemes and absconding with the money. The existence of a large number of financially "illiterate" investors (Lusardi and Mitchell, 2014) opens the door for financial fraud because these investors are likely to be attracted by products that offer too-good-to-be-true returns.[1] Misleading product descriptions may induce naive investors to underestimate default risk and purchase products that are not consistent with their risk attitudes. The spread of financial fraud suggests that many naive investors may be unaware of the possibility of such fraud. To prevent firms from exploiting these naive investors, policymakers may employ regulatory policies in financial markets such as interest rate ceilings,[2] restrictions on product design,[3] and minimum legislative standards for firms.[4] However, excessive regulations may limit the product choices of investors and possibly reduce welfare. Therefore, the level of sophistication possessed by general investors is an important factor in determining whether certain regulations are necessary.

Given this background, we build a model with firm(s) strategically choosing whether to exploit naive investors by offering financial products with too-good-to-be-true returns. There are a fraction of naive investors who are unaware that a firm can commit financial fraud. Specifically, they do not know that the firm can seize the returns on their investment, and thus underestimate the true risk of a fraudulent financial product. Therefore, naive investors' investment decisions are inconsistent with their risk attitudes. Their behaviors, in turn, create an incentive for the firm to commit financial fraud.

After studying the monopoly case, we introduce an entrant firm to the market and consider a leader-follower model. With competition, three types of equilibrium may arise depending on the costs of committing financial fraud. When the costs are high, both firms offer normal products, but their profits will be driven down to zero due to price (rate of return) competition. When the costs are low, both firms offer fraudulent products with returns at the interest rate ceiling, and they share the market for naive investors. In between, the market falls into a separating equilibrium: One firm offers a high-return fraudulent

[1]A few papers have attempted to theoretically study the link between financial literacy and investment decisions, as well as the (mis-)selling behaviors of financial professionals in the market (e.g., Lusardi et al., 2017).

[2]Many countries have imposed interest rate ceilings (`https://en.wikipedia.org/wiki/Interest_rate_ceiling`). See Modigliani and Sutch (1966) for a discussion.

[3]For example, the Investment Company Act of 1940 strictly regulates the structure of mutual funds, imposing severe restrictions on liabilities and complex capital structures. See Campbell et al. (2010) for an extensive discussion.

[4]One example of such policy is the SAFE Mortgage Licensing Act passed in 2008. See `https://mortgage.nationwidelicensingsystem.org/SAFE/Pages/default.aspx`

product that attracts all naive investors, while the other firm sells a normal product to all sophisticated investors. In this case, both firms earn positive profits. As a result, if the market leader offers a normal product in a monopoly, investor welfare can be harmed by the follower's entry, because naive investors will be exploited by the fraudulent product in the separating equilibrium. Interestingly, we show that the leader has a stronger incentive to commit financial fraud than in the monopoly case, because a competing entrant firm makes it less attractive to offer a normal product. Our main results of market segregation and competition triggering financial fraud are robust after introducing endogenous entry, multi-product firms, and more than two firms to the model.

Next, we study the effect of information disclosure that reduces the proportion of naive investors. In the baseline model with a monopolistic firm, policymakers can compel the firm to behave honestly through information disclosure. Moreover, if the proportion of naive investors drops below a certain threshold, the firm will not offer fraudulent products even if doing so is costless because of the fear of losing sophisticated investors.

In the leader-follower model, we study firms' private incentive to disclose or unshroud information as well as its policy implications. Suppose that each firm can costlessly disclose the information about the possibility of financial fraud, which reduces the proportion of naive investors. We find a trade-off in the case of separating equilibrium: The honest firm has the incentive to increase the proportion of sophisticated investors to obtain a larger market share. However, it does not want to increase this proportion too much because if exploiting naive investors becomes unprofitable, the other firm would deviate from offering a fraudulent product and start competing for sophisticated investors. Under this trade-off, while lowering the interest rate ceiling makes the fraudulent product less attractive, it may not be welfare-improving if the honest firm strategically conceals information to prevent the dishonest firm from competing in the market for normal products. Similarly, increasing legal punishment and implementing a public education program also discourage the honest firm from disclosing information.

1.1 Related literature

Our paper contributes to the growing literature of bounded rationality and its applications to industrial organization and contracting problems. First, depart from the extensive literature of shrouded attributes and add-on pricing (e.g., Gabaix and Laibson, 2006; Agarwal et al., 2017; Heidhues et al., 2017; Kosfeld and Schüwer, 2017), we consider two types of products. While the fraudulent product can be considered as having shrouded side effects, the normal product is by nature a "transparent" product, with no add-on or shrouded attribute. Such product differentiation and firms' cost heterogeneity give us the market segmentation result, which is not presented in most of the add-on pricing papers such as Gabaix and Laibson (2006) and Heidhues et al. (2017). Even for papers with similar results, their mechanisms are different from ours. For instance, in Herweg and Rosato (2020), market segregation is driven by firms facing different degrees of agency frictions, and in Armstrong et al. (2009), it comes from consumers having heterogeneous search costs.

Second, we consider a leader-follower model that has not yet been fully studied, and extend it to allow for costly entry. Our results that firms have limited incentive to disclose information are exactly driven by the dynamic nature of the game: The market leader does not want to increase the share of sophisticated investors too much, since it will attract the follower to compete with the leader in the second stage. In the literature of consumer obfuscation, Carlin (2009) shows that firms strategically increase their price complexity as a response to intensified competition. In the literature of product quality, Armstrong and Chen (2009) find that firms may cheat inattentive consumers that ignores product quality, and a market transparency policy which boosts the number of attentive consumers will make firms less inclined to cheat and improve welfare.[5] All these papers document some static environments in which competition fails to benefit consumers, which differentiate them from our dynamic approach.

Third, in our paper the perverse welfare effect of policy intervention comes from crowding out the firm's private incentive to disclose information. Recent works like Murooka and Schwarz (2018) and Johnen (2019) also show that choice-enhancing policies that make it easier for consumers to switch can decrease consumer and social welfare if the firms adjust their pricing strategies in response, but their mechanisms are not through unshrouding. Li et al. (2016) find that mandatory disclosure policies may discourage the firm's investment and harm consumers. In their model, investment is costly, while in the present paper, firms have limited incentive to disclose even if doing so is costless. More generally, in a review paper, Spiegler (2015) documents papers showing firms' change of product quality in response to the mandatory disclosure policy together with other unintended consequences. In the present paper, however, firms respond to policies by changing their decisions on information disclosure rather than products.

Finally, the paper also adds to the study of contract theory with boundedly rational agents. Naive investors in our framework can be broadly understood as overconfident (Grubb, 2009), inattentive (Armstrong and Chen, 2009), or neglecting financial product issuers' informational advantage (Kondor and Kőszegi, 2017). Our model can further be explained as

[5] See, e.g., Dranove and Jin (2010) for a recent survey on the literature of vertically differentiated products and information disclosure.

a moral hazard problem with agents unaware of some actions (von Thadden and Zhao, 2012; Auster and Pavoni, 2020, 2021) or states (Auster, 2013; Lei and Zhao, 2021).[6]

The paper proceeds as follows. In Section 2, we lay out the basic setting of the interaction between a monopoly firm and a representative investor who is possibly unaware that the firm may commit financial fraud. Section 3 presents our main results regarding market segregation and competition under various settings. Section 4 studies information disclosure and implications of policy instruments. Section 5 concludes. Proofs are relegated to Appendix C.

2 Baseline Model with a Monopolist

We start from the simple case of monopoly. A monopoly firm in this market needs outside finance $I > 0$ to fund a risky project that may succeed with probability p and fail with probability $1 - p$. If the project succeeds, it generates a positive cash return $y = R$ for the firm. If it fails, the return is normalized to $y = 0$. The outcome of the project y is publicly observable. A representative investor (he) has initial wealth $\omega > I$, with an isoelastic (CARA) utility function

$$u(m) = \begin{cases} \frac{m^{1-\alpha}}{1-\alpha} & \text{when } \alpha \geq 0 \text{ and } \alpha \neq 1, \\ \ln m & \text{when } \alpha = 1. \end{cases}$$

where m is the net payoff in a contingency. The risk preference parameter α is distributed over $[0, +\infty)$ with a commonly known c.d.f. $F(\alpha)$ and a strictly positive p.d.f. $f(\alpha)$. We assume that the distribution of α satisfies the standard monotone likelihood ratio property (MLRP), namely, $f(\alpha)/[1 - F(\alpha)]$ is nondecreasing in α.

The firm offers a financial product as a contract that specifies a monetary return $r \leq \overline{R}$ repaid to the investor when $y = R$ and zero otherwise. The upper bound $\overline{R}$ represents the interest rate ceiling permitted by law. In addition to r, the firm also chooses whether to behave honestly ($x = 0$) or commit financial fraud ($x = 1$). If the firm chooses $x = 0$, it repays r to the investor as stated in the contract when $y = R$. We call the contract with $x = 0$ a *normal product*. If the firm chooses $x = 1$, it repays nothing to the investor when the project succeeds and absconds with the total return R. We call the contract with $x = 1$

[6] Relatedly, when there is adverse selection, Francetich and Schipper (2020) study a procurement problem in which the principal is unaware of events affecting the agent's marginal costs. When contracts are incomplete, small shocks can cause significant asset-price volatility, which may lead to severe financial fraud. For an intensive literature review, see Allen and Gale (2005).

a *fraudulent product*. Providing a fraudulent product incurs a cost $c > 0$ to the firm, which can be interpreted as the expected reputation loss or legal punishment. Therefore, the firm's private benefit from committing financial fraud is R when the project succeeds. Suppose that $pR > I$, so the project is *ex ante* profitable for the risk-neutral firm.

Figure 1: Timeline in the Baseline Model

The timing of the game is depicted in Figure 1 and described as follows. The firm first chooses x and offers a contract with repayment r to the investor. If the investor rejects the offer, both parties receive zero payoffs and the game ends. If the investor accepts the offer, y is realized and observed. Given that $y = R$, the firm repays r if $x = 0$ and zero if $x = 1$. Note that the firm decides whether to commit financial fraud before the realization of the project outcome, because it involves certain preparations before offering the fraudulent product to the investor.[7]

When the investor observes that the firm is behaving honestly ($x = 0$), he accepts the firm's offer if

$$p\frac{(\omega + r)^{1-\alpha}}{1-\alpha} + (1-p)\frac{(\omega - I)^{1-\alpha}}{1-\alpha} \geq \frac{\omega^{1-\alpha}}{1-\alpha}.$$

We denote the solution to this inequality by $\alpha \leq \overline{\alpha}(r)$. It can be verified that $\overline{\alpha}(r)$ is increasing in p and r, which implies that an investor is more likely to invest in a project with lower default risk and a higher rate of return.

An honest firm chooses r to maximize its expected profit, i.e., $p(R - r)F(\overline{\alpha}(r))$. The first-order condition is

$$-pF(\overline{\alpha}(r)) + p(R - r)f(\overline{\alpha}(r))\overline{\alpha}'(r) = 0. \tag{1}$$

When $r = I$, the left-hand side of (1) is positive; when $r = R$, the left-hand side of (1) is negative. Hence, under the MLRP, there is an interior solution $r^* \in (I, R)$ that yields a

[7] For example, the firm has to prepare exaggerated marketing materials to attract potential investors and/or provide a forged auditor's report to deceive the regulators. Detailed descriptions of these preparations can be found in many court cases. See, e.g., United States of America vs. Robert A. Stanford (https://im.ft-static.com/content/images/b6eabf92-b631-11e1-8ad0-00144feabdc0.pdf).

profit $p(R - r^*)F(\overline{\alpha}(r^*))$ to the firm. We assume that $\overline{R}$ is sufficiently large so that $\overline{R} > r^*$. If $\overline{R} \leq r^*$, the firm will simply choose $\overline{R}$ since its profit function is nondecreasing when $r \in (0, r^*)$. Note that setting $\overline{R} < r^*$ hurts investor welfare as the binding interest rate ceiling prevents the firm from offering its optimal product and distorts the market.

Now we are ready to introduce financially illiterate investors into the model. Suppose that there are two types of investors in the population. A fraction $\lambda \in (0, 1)$ of investors are *naive*: They are unaware that the firm has the option $x = 1$. In other words, they mistakenly believe that the firm can only choose $x = 0$ and do not have financial fraud in mind.[8] The remaining fraction $1 - \lambda$ of investors are *sophisticated*, as they are fully aware of and can observe the firm's action x. This observability assumption echoes the literature of product quality and consumer search, such as Armstrong and Chen (2009) which assumes that some consumers can directly observe the quality of firms.

We adopt the solution concept of generalized Nash equilibrium defined by Halpern and Rêgo (2014) for extensive games with unaware players. This solution concept is also compatible with the notion *generalized extensive-form games* proposed in Heifetz et al. (2013). In Appendix A, we show that our benchmark model can be reinterpreted in the language of Heifetz et al. (2013).

For expositional clarity, let

$$\pi^0 = p(R - r^*)F(\overline{\alpha}(r^*)),$$
$$\pi^1 = pRF(\overline{\alpha}(\overline{R})).$$

Here, π^0 is the firm's revenue when it provides a normal product ($x = 0$); π^1 is the firm's revenue when it provides a fraudulent product ($x = 1$). Because $\overline{\alpha}(r)$ is decreasing in r and $r^* < \overline{R}$, we have $\pi^0 < \pi^1$. From the firm's perspective, selling a normal product is less profitable than selling a fraudulent product to exploit naive investors.

Let (r, x) denote the firm's pure strategy. From our previous analysis, when the firm chooses $x = 0$, it will offer $r = r^*$ to maximize its expected profit. When the firm chooses $x = 1$, it will propose $r = \overline{R}$ to attract as many naive investors as possible. Therefore, the

[8]Our assumption of investor awareness is motivated by findings in a companion paper Gui et al. (2021). Based on experimental evidence, Gui et al. (2021) find that a large fraction of investors make investment decisions that are inconsistent with their risk preferences, implying that they may be unaware of the high risk associated with high-return products. Besides, we assume that naive investors are unaware that they may be unaware. That is, when they see the firm offering an unreasonably high rate of return, such as $r = \overline{R}$, they will simply take it as a mistake without further reasoning. Some other works, e.g., Chung and Fortnow (2016); Tirole (2009, 2016); Zhao (2015), assume that agents recognize that their cognitive ability is limited and that their understanding of the game could be incorrect. In other words, agents are aware of their unawareness.

firm plays $(r^*, 0)$ if

$$\pi^0 \geq \lambda\pi^1 - c \;\Leftrightarrow\; c \geq \lambda\pi^1 - \pi^0 \equiv c^*.$$

The results above are summarized in Proposition 1.

Proposition 1. *There exists $c^* = \lambda\pi^1 - \pi^0$ such that:*

(a). When $c \geq c^*$, there exists an equilibrium in which the firm plays $(r^*, 0)$, i.e., it offers a normal product with a rate of return r^*; both types of investors with $\alpha \leq \overline{\alpha}(r^*)$ accept the contract.

(b). When $c \leq c^*$, there exists an equilibrium in which the firm plays $(\overline{R}, 1)$, i.e., it offers a fraudulent product with a rate of return $\overline{R}$; sophisticated investors reject the contract, while naive investors with $\alpha \leq \overline{\alpha}(\overline{R})$ accept the contract.

Note that, if the firm proposes $r = r^*$, the default risk is $1-p$; if the firm proposes $r = \overline{R}$, the default risk is 1. Therefore, a high-return financial product is associated with a high risk of default as a result of financial fraud.

It is also instructive to compute investors' welfare change ΔW when the firm offers different products. When a normal product (N) is provided, investors' welfare change is

$$\Delta W_N^m = \int_0^{\overline{\alpha}(r^*)} \frac{p(\omega + r^*)^{1-\alpha} + (1-p)(\omega - I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha), \tag{2}$$

where the superscript m represents "monopoly". When a fraudulent product (F) is provided, investors' welfare change is

$$\Delta W_F^m = \lambda \int_0^{\overline{\alpha}(\overline{R})} \frac{(\omega - I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha). \tag{3}$$

Clearly, $W_F^m < W_N^m$.

3 Model with Multiple Firms

We extend the baseline model to account for competition between heterogeneous firms. We start with a leader-follower model to establish our main results. We then show that the results are robust in the cases of endogenous entry and more than two firms.

3.1 A leader-follower model

There is mounting evidence that the reputation or credibility of financial intermediaries depends on their historical performance, so well-established firms may suffer huge reputation loss after their financial misconduct gets detected (Chemmanur and Fulghieri, 1994; Gopalan et al., 2011; Griffin et al., 2014). Besides, CEOs and board members get financial penalties or reputation losses after financial fraud, and these penalties are related to firm-specific factors such as firm size and corporate governance (Agrawal et al., 1999; Fich and Shivdasani, 2007). Therefore, well-established firms may have a larger cost of committing financial fraud, because they either have good performance records in the past or attract substantially more attention from regulatory authorities, while newcomers are likely to care less about their reputation. In this section, we model the incumbent firm as the market leader with a (possibly) higher cost of committing financial fraud and introduce an entrant acting as the follower that can observe the leader's action and respond strategically.[9] We are also interested in their incentive to disclose information about the possibility of financial fraud. The analysis provides several insights into regulation and policies.

Consider the same environment as in the baseline model except that now there are two firms in the market: a leader and a follower. Because the leader is the incumbent, it is natural to assume that it faces a higher cost of committing financial fraud than the follower. For convenience, denote the leader as firm H who faces cost c_H; denote the follower as firm L who faces cost c_L. Assume that $0 < c_L \leq c_H$. We take firm L's entry as given and assume that entry is costless.

Investors are wealth-constrained, so each investor can purchase only one product. This assumption implies that, *ceteris paribus*, investors will purchase whichever product offers a higher rate of return. If an investor is indifferent between two products, he purchases each product with equal probability.

The timing of the game is now depicted in Figure 2 and described as follows. First, firm H chooses whether to commit financial fraud (x_H) and the rate of return of its product (r_H). Observing firm H's decision, firm L chooses whether to commit financial fraud (x_L) and the rate of return of its product (r_L). Then, investors decide whether and which product to purchase. Finally, the outcomes of both projects are realized and payments are made according to contracts.

Depending on c_H and c_L, there are several types of equilibrium based on x_H and x_L. In

[9] The leader-follower setting echoes the observation that the financial sector is usually less competitive, and there are several large firms or banks acting as market leaders. For example, Nathan and Neave (1989) find no decreasing trend in asset concentrations in Canada's financial system. Bikker and Haaf (2002) show that the banking industry is highly concentrated in countries like Denmark, Greece, Netherlands, and Switzerland.

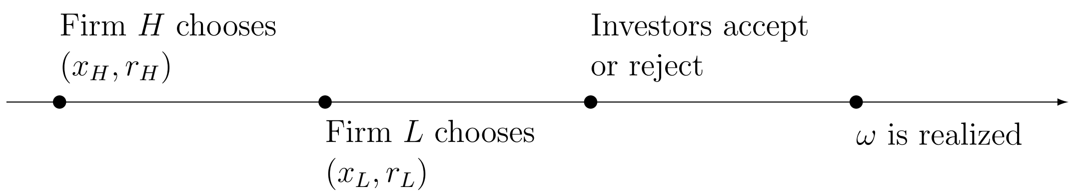

Figure 2: Timeline in the Leader-follower Model

a *normal-product equilibrium* (NE), both firms offer normal products ($x_H = x_L = 0$). In a *fraudulent-product equilibrium* (FE), both firms commit financial fraud ($x_H = x_L = 1$). Finally, in a *separating equilibrium* (SE), one firm provides a normal product, while the other firm commits financial fraud. For simplicity, we focus on the case with a nonbinding interest rate ceiling, i.e., $\overline{R} > R$.

Proposition 2 summarizes the equilibrium of the leader-follower model.

Proposition 2. *There exists $c_L^* = \lambda\pi^1$ and $c_H^* = \lambda\pi^1/2 - (1-\lambda)\pi^0$ such that:*

(a). When $c_L \geq c_L^*$, there exists an NE in which both firms offer a normal product with $r = R$.

(b). When $c_L \leq c_H^*$ and $c_H \leq c_H^* + (1-\lambda)\max\{0, c_L - c^*\}$, there exists an FE in which both firms offer a fraudulent product with $r = \overline{R}$.

(c). When $c_L \leq c_L^*$, and either $c_L \geq c_H^*$ or $c_H \geq c_H^* + (1-\lambda)\max\{0, c_L - c^*\}$, there exists an SE in which one firm offers a normal product, while the other firm offers a fraudulent product with $r = \overline{R}$.

In sum, when both firms have high costs of committing financial fraud, they behave honestly and an NE arises. When both firms have low costs, they offer fraudulent products and an FE arises. SE occurs between these two cases.

There are two remarks regarding Proposition 2. First, $c_H^* > 0$ if and only if $\lambda > \overline{\lambda} \equiv 2\pi^0/(2\pi^0 + \pi^1)$. When $\lambda < \overline{\lambda}$, there is no FE. In other words, when the fraction of naive investors are too small, there is no such equilibrium that both firms commit financial fraud, as the profits from selling fraudulent products are limited.

Second, the equilibrium conditions in (b) and (c) of Proposition 2 can be further simplified, depending on the relationship between π^0 and π^1. In particular, when $\pi^1 \geq 2\pi^0$, we have: (i) If $c_L \geq c_L^*$, there exists an NE; (ii) If $c_H \leq c_H^*$, there exists an FE; (iii) If $c_L \leq c_L^*$ and $c_H \geq c_H^*$, there exists an SE. Figures 3 and 4 illustrate two cases for the results of Proposition 2.

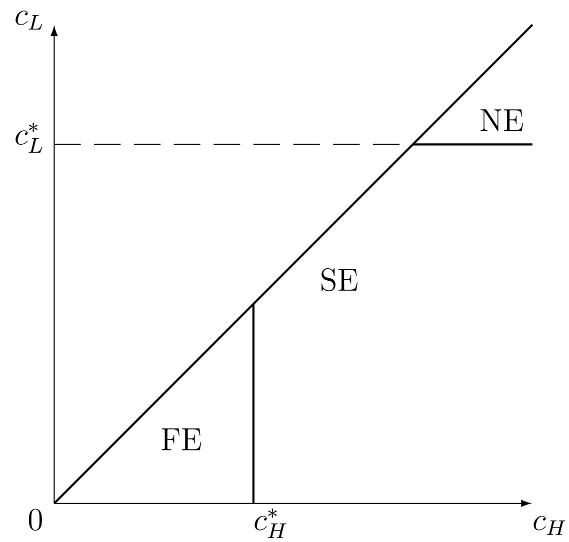

Figure 3: Graphical illustration of Proposition 2 ($\pi^1 \geq 2\pi^0$)

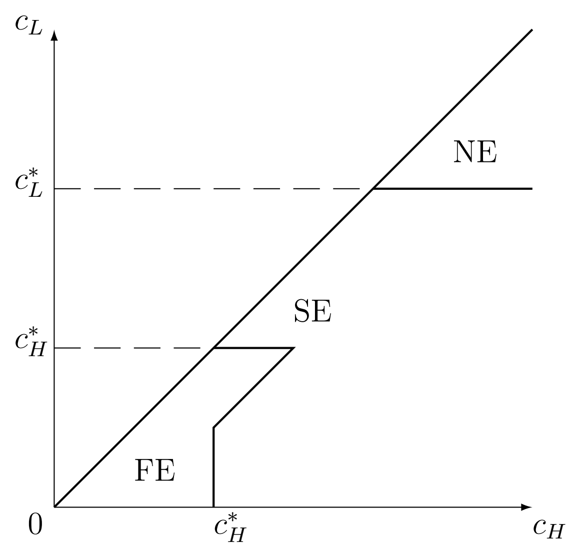

Figure 4: Graphical illustration of Proposition 2 ($\pi^1 < 2\pi^0$)

It is worth noting that, although facing more severe punishment than the follower, the market leader may commit financial fraud in an SE. According to the proof of Proposition 2, this is the case when $c_H^* \leq c_L \leq c_L^*$ and $c_H \leq c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$. By comparing these conditions with c^* in Proposition 1 for the case of monopoly, we can determine whether having firm L in the market changes firm H's incentive to commit financial fraud. Besides, we can also discuss whether firm L's presence affects investor welfare. Let ΔW_{NE}^c, ΔW_{FE}^c, ΔW_{SE}^c denote investors' welfare change when the equilibrium is NE, FE, SE, respectively. The superscript "c" represents "competition". The results are summarized in Proposition 3.

Proposition 3. *(a). $c^* < c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$. That is, firm H has a stronger incentive to commit financial fraud in a leader-follower market than in a monopoly. Having firm L in the market exacerbates financial fraud.*

(b). Suppose that firm H plays $(r^*, 0)$ in an SE. Then, $\Delta W_{FE}^c = \Delta W_F^m < \Delta W_{SE}^c < \Delta W_N^m < \Delta W_{NE}^c$. In other words, having firm L in the market will make investors worse off.

Intuitively, a monopolistic firm can sell its product to the whole market; however, in a leader-follower environment, firms earn zero profit in the NE. If any of them deviates to offer a fraudulent product, it immediately obtains a positive profit that is the same as that of a monopoly, as it can capture most of the naive investors. Therefore, selling a normal product becomes undesirable, while selling a fraudulent product turns to be attractive to the leader when a follower is present.

Proposition 3 suggests that opening the monopoly market may harm investors: If the incumbent firm offers a normal product in a monopoly, while the entrant offers a fraudulent product and the market equilibrium is an SE or FE, then investors may be worse off after firm L's entry. Intuitively, having a follower drives each firm's profit in an NE down to zero, making it easier for firms to deviate and offer a fraudulent product. The welfare loss comes from naive investors who purchase normal products in a monopoly but get exploited by fraudulent products in an SE.

Our findings echo several theoretical and empirical results in the literature. Shleifer (2004) provides several examples in which unethical conduct emerges as a result of market competition. The key idea is that unethical conduct sometimes reduces costs or raises revenues, so firms may engage in censured behaviors as a response to intensified competition. This intuition is similar to that of Proposition 3, but we provide a formal model of this effect. Relatedly, Ru and Schoar (2016) find that less-sophisticated households are much more likely to be offered credit cards with back-loaded or hidden fees as a result of the screening strategies implemented by credit card companies, but this paper does not specifically focus on competition between firms. Agarwal et al. (2017) document that deregulation and competition increase the proportion of naive borrowers and, thus, may change the former unshrouded-price equilibrium into a shrouded-price equilibrium. Di Maggio et al. (2019) find that deregulation intensifies competition and increases the supply of more complex and risky mortgages.

3.2 Endogenous entry

In the leader-follower model, firm L's entry is taken as given. Now, we relax this assumption and study firm L's entry decision when it has to incur a positive fixed cost $\kappa > 0$ to enter the market.

Proposition 4. *Suppose that $\pi^1 > 2\pi^0$ and $\lambda \geq \overline{\lambda} = \frac{2\pi^0}{2\pi^0+\pi^1}$.*

(a). When $\kappa \leq (1-\lambda)\pi^0$, firm L's entry is deterred if $c_L + \kappa > c_L^*$ and $c_H \geq c^*$. In this case, firm H offers a normal product.

(b). When $\kappa > (1-\lambda)\pi^0$, firm L's entry is deterred if $c_L + \kappa > c_L^*$, or $\frac{c_L^*}{2} < c_L + \kappa \leq c_L^*$ and $c_H \leq c^* + \lambda\pi^0$. In this case, firm H offers a normal product if $c_L + \kappa > c_L^*$ and $c_H \geq c^*$; otherwise it offers a fraudulent product.

(c). When $c_L + \kappa \leq \frac{c_L^*}{2}$ and $c_H \leq c_H^*$, firm L's entry is accommodated, and there exists an FE.

(d). Otherwise, firm L's entry is accommodated, and there exists an SE. Firm H offers a normal product if $c_H \leq c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$.

Figure 5 gives a graphical illustration of Proposition 4 with a sufficiently large entry cost, i.e., $\kappa > (1-\lambda)\pi^0$.

According to Proposition 4, a higher entry cost results in less entry. To see this, note that the two cutoffs for c_L are $c_L^* - \kappa$ and $c_L^*/2 - \kappa$. They all decrease with κ. Moreover, entry is jointly determined by the entry cost κ and the follower's cost of committing financial fraud c_L. These two costs are complimentary.

However, the effect of entry cost on firms' incentive to commit financial fraud is not monotone. A higher entry cost does not necessarily imply more or less financial fraud. For instance, the incumbent firm H offers a normal product in a monopoly whenever $c_H \geq c^*$. But in the presence of a potential entrant, firm H's product choice relies on $c_L + \kappa$. In Corollary 1, we take a subset of parameter values of c_L and c_H as an example and show that as κ increases, firm H's product choice switches back and forth between normal and fraudulent products.

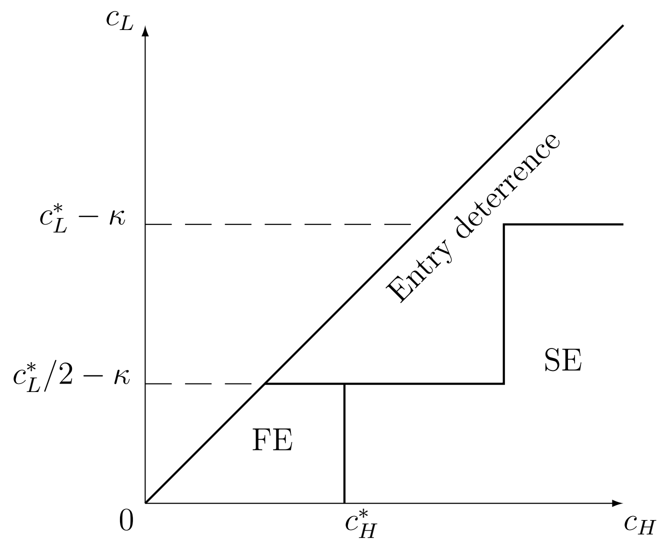

Figure 5: Graphical illustration of Proposition 4 ($\kappa > (1-\lambda)\pi^0$)

Corollary 1. *Suppose that $\pi^1 > 2\pi^0$ and $\lambda \geq \overline{\lambda}$. Consider the situation with $c_L \leq c_H^*$ and $c^* \leq c_H \leq c^* + \lambda\pi^0$.*

(a). When $\kappa \leq \frac{c_L^*}{2} - c_L$, firm L's entry is accommodated, and firm H offers a normal product.

(b). When $\frac{c_L^*}{2} - c_L < \kappa \leq c_L^* - c_L$, firm L's entry is deterred, and firm H offers a fraudulent product.

(c). When $\kappa > c_L^* - c_L$, firm L's entry is deterred, and firm H offers a normal product.

Intuitively, when κ is sufficiently small, it is too costly for the incumbent to strategically deter entry. Therefore, firm H will offer a normal product due to its high cost of committing financial fraud. When κ increases to an intermediate level, the incumbent wants to protect its monopoly power, thus switches to committing financial fraud. Since c_L is small, firm L prefers to offer a fraudulent product. But firm H's deviation to fraudulent products reduces firm L's profit upon entry. When κ is sufficiently large, firm L has no incentive to enter at all. Firm H therefore offers a normal product again, which is its optimal choice as a monopolist.

Investors' welfare change ΔW is again a non-monotonic function of the entry cost. It is worth mentioning that there is no NE when the entry cost is positive since both firms earn zero profit in an NE. Therefore, naive investors are free from exploitation only when firm H offers a normal product to deter entry. To see this formally, consider again the example in Corollary 1: When $\kappa \leq c_L^*/2 - c_L$, the market equilibrium is an SE. Naive investors get exploited, while sophisticated investors benefit from purchasing normal products. When $c_L^*/2 - c_L < \kappa \leq c_L^* - c_L$, the market is a monopoly with only fraudulent products, hence investor welfare is lower than that of an SE. When $\kappa > c_L^* - c_L$, the market is a monopoly with only normal products, hence investor welfare is higher than that of an SE. Therefore, simply increasing or decreasing the entry barrier for low-cost firms may not guarantee a welfare improvement for investors.

3.3 More than two firms

The leader-follower model can also be extended to allow for more than two firms. Consider the same environment as in the baseline model except that now there are m firms in the market: k leaders and $m-k$ followers, where $m > k \geq 1$. We denote firms by $j = 1, \ldots, m$, and each firm's cost of committing financial fraud by c_j. Firms with $1 \leq j \leq k$ are leaders, and the remaining firms are followers. We also assume that firms only differ in their costs of committing financial fraud and are identical in all other aspects. In particular, $c_1 \geq c_2 \geq \cdots \geq c_m \geq 0$. That is, leaders have higher costs of committing financial fraud than followers. We maintain the assumption that firms compete in the rate of returns, implying that *ceteris paribus*, investors will purchase whichever product that offers a higher rate of return. If an investor is indifferent between two products, she purchases each product with equal probability.

The timing of the game is now described as follows. First, leaders simultaneously choose whether to commit financial fraud ($x_j,\ 1 \leq j \leq k$) and the rates of return of their products ($r_j,\ 1 \leq j \leq k$). Observing leaders' decisions, followers simultaneously choose whether to commit financial fraud ($x_j,\ k+1 \leq j \leq m$) and the rates of return of their products ($r_j,\ k+1 \leq j \leq m$). Then, investors decide whether and which product to purchase. Finally, the outcomes of all firms' projects are realized and payments are made according to contracts.

We use (q_L, q_F) to represent an equilibrium in which there are q_L leading firms and q_F following firms providing normal products. It is without loss to focus on the case that firms 1,...,q_L (when $q_L > 0$) and $k+1, ..., k+q_F$ (when $q_F > 0$) offer normal products. That is, normal products are produced by leaders and followers with the highest costs. The parameter space supporting other cases should be more restrictive than the ones discussed here. Our results are formally stated in Proposition 5.

Proposition 5. *Define $\overline{c} = \lambda\pi^1$ and $\underline{c} = \lambda\pi^1/m - (1-\lambda)\pi^0$. Let the number of leading firms and following firms offering normal product, (q_L, q_F), represent an equilibrium.*

(a). There exists an equilibrium $(0,0)$ *if and only if*

$$c_{k+1} \leq \underline{c},\ c_1 \leq \underline{c} + (1-\lambda)\max\{0, c_{k+1} - \frac{\bar{c}}{m-1} + \pi^0\}.$$

(b). There exists an equilibrium $(1,0)$ *if and only if either*

$$c_{k+1} \leq \underline{c},\ (m-1)c_2 \leq \bar{c} \leq m[c_1 + (1-\lambda)\pi^0],$$

or

$$c_{k+1} \geq \underline{c},\ (m-1)c_2 \leq \bar{c} \leq (m-1)[c_1 + (1-\lambda)\pi^0].$$

(c). There exists an equilibrium $(0,1)$ *if and only if*

$$c_{k+1} \geq \underline{c},\ (m-1)c_{k+2} \leq \bar{c} \leq (m-1)c_{k+1}.$$

(d). For any nonnegative integers q_L *and* q_F *such that* $1 < q_L + q_F = q < m$*, there exists an equilibrium* (q_L, q_F) *if and only if either*

$$(m-q+1)c_{k+1+q_F} \leq (m-q)c_{q_L+1} \leq \bar{c} \leq (m-q+1)c_{k+q_F},$$

or

$$(m-q)c_{q_L+1} \leq \bar{c} \leq \min\{(m-q)c_{q_L}, (m-q+1)c_{k+1+q_F}\}.$$

(e). There exists an equilibrium $(k, m-k)$ *if and only if* $c_m \geq \bar{c}$*.*

Hence, when there are more than two firms in the market, NE (part (a) of Proposition 5) and FE (part (e) of Proposition 5) become special cases of separating equilibria. Similar to Proposition 2, equilibria in Proposition 5 are determined by the heterogeneity of costs of committing financial fraud.

3.4 Multi-product firms

One of the key assumptions we posit in the leader-follower model is that each firm can only offer a single type of product, because if the fraud is caught, the reputation loss or the punishment will prevent the firm from offering normal financial product. In this section, we relax this assumption and allow for multi-product firms.

Proposition 6. *(a). In a monopoly, the firm always sells a normal product with a rate of return* r^**, and sells a fraudulent product with a rate of return* $\overline{R}$ *if and only if* $c \leq \lambda(\pi^1 - \pi^0)$*.*

(b). In a leader-follower duopoly, both firms sell normal products with a rate of return R*. When* $c_H \leq \lambda\pi^1/2$*, both firms offer fraudulent products. When* $c_H > \lambda\pi^1/2$ *and* $c_L \leq \lambda\pi^1$*, only firm* L *offers a fraudulent product.*

By Proposition 6, in a multi-product market, firms always produce normal products as they are costless. Therefore, the only question is when do firms offer fraudulent products.

Comparing part (a) of Proposition 6 with Proposition 1, we have $c^* < \lambda(\pi^1 - \pi^0)$. Comparing part (b) of Proposition 6 with Proposition 2, we have $c_H^* < \lambda\pi^1/2 < c_L^*$. That is, multi-product firms are more likely to offer a fraudulent product.

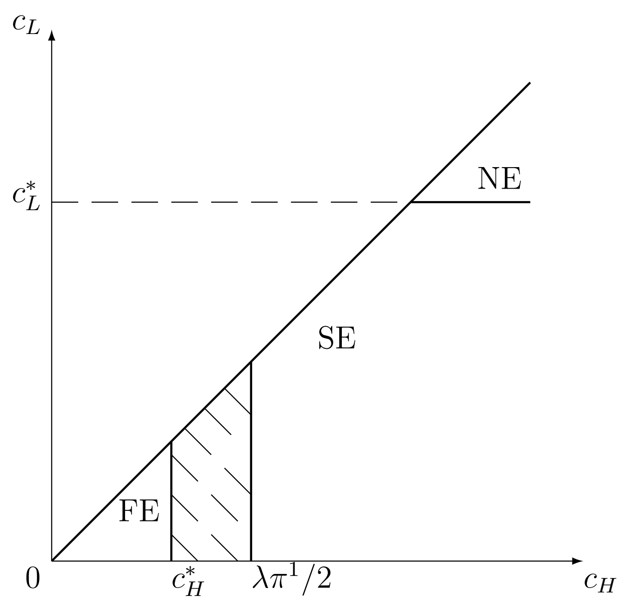

Figure 6: Graphical illustration of part (b) of Proposition 6

Figure 6 combines Figure 3 with part (b) of Proposition 6. In the NE region, no firm offers fraudulent product, this is identical to our results in Proposition 2. In the SE region, only firm L offers a fraudulent product. In the FE region, both firms offer fraudulent products. In Proposition 2 the SE region and FE region are split by the cutoff c_H^*, while with multi-product firms, the two regions are now split by the cutoff $\lambda\pi^1/2 > c_H^*$. As a result, in the shaded area of Figure 6, the single-product market has only one firm offering a fraudulent product, whereas in the multi-product market both firms commit financial fraud.

Intuitively, when a monopolist can offer multiple products, it is not afraid of losing sophisticated investors when offering a fraudulent product. Therefore, the opportunity cost of committing financial fraud decreases. In a leader-follower duopoly, multi-product firms always compete for sophisticated investors, which drives the profit of normal products down to zero. This also reduces their costs of committing financial fraud.

4 Discussion and Policy Implication

4.1 Information disclosure in a monopoly

Consider a public education or information disclosure program that raises awareness of financial fraud and thus reduces the fraction of naive investors. Based on the monopoly model in Section 2, our observation is that information disclosure not only directly prevents some of the naive investors from being mis-sold, but also indirectly reduces or even eliminates financial fraud. According to Proposition 1, the monopoly firm commits financial fraud if the cost c is below the cutoff c^*, and this cutoff is increasing in λ. Therefore, having more naive investors triggers the firm to offer a fraudulent financial product. In contrast, if information disclosure reduces λ, thereby lowering the cutoff cost c^*, then the firm will more often find it less worthwhile to commit financial fraud. Moreover, when λ is sufficiently low, i.e., $\lambda \leq \pi^0/\pi^1$, c^* becomes nonpositive, and thus the firm has no incentive to commit financial fraud at all. This result is summarized in Corollary 2, with graphical illustration in Figure 7.

Corollary 2. *c^* increases with λ, and when $\lambda \leq \pi^0/\pi^1$, $c^* \leq 0$. That is, a decrease in λ implies a decrease in the firm's incentive to commit financial fraud. When λ is sufficiently small, the firm has no incentive to commit financial fraud irrespective of c.*

The comparative statics in Corollary 2 leads to a common result in the behavioral contracting literature: Information disclosure not only prevents some naive investors from being exploited but also reduces the firm's incentive to offer a fraudulent product.[10] The firm is more likely to behave honestly in a market with a larger proportion of sophisticated investors.[11]

[10]Relatedly, von Thadden and Zhao (2012) and Li et al. (2016) find that monopolists tend to disclose the adverse effects of their products when the market is dominated by sophisticated consumers. Zhou (2008) suggests that naive consumers tend to overestimate the importance of certain attributes of a product after firms' advertising and finds that increasing the proportion of naive consumers will also reduce the surplus of sophisticated consumers. Schumacher and Thysen (2021) provide a synthetic model with this self-reinforcing pattern in a more general setting.

[11]Our result in Corollary 2 is in line with Jin and Leslie (2003) offering empirical evidence that mandatory information disclosure increases restaurants' hygiene scores by making consumers aware of the health risks. That is, information disclosure may have both direct effects (i.e., it increases consumers' awareness of health risks) and indirect effects (i.e., as more consumers become aware of health risks, restaurants have higher incentive to improve their hygiene scores).

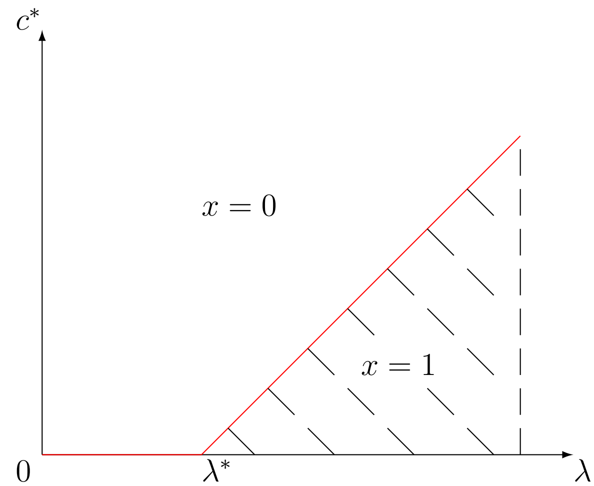

Figure 7: Graphical illustration of Corollary 2

4.2 Information disclosure in a competitive market

Based on the leader-follower model in Section 3.1, we study firms' incentive to disclose information. Suppose that firms can costlessly disclose information (educate or unshroud) to investors about the fact that $x = 1$ is included in their choice sets when making offers to investors. We assume that the fraction of naive investors will change from λ to $\lambda - \delta$ ($\delta \in (0, \lambda)$) as long as one of the two firms decides to unshroud.

We restrict our attention to the combinations of c_L and c_H that lead to an SE (without information disclosure), which is a more prevalent scenario because most financial markets consist of both honest firms and fraudulent firms.[12]

Firm L offering a fraudulent product has no incentive to unshroud. Firm H, which behaves honestly, may have an incentive to unshroud because doing so increases the proportion of sophisticated investors. As investors become aware of the possible financial fraud, they will stop purchasing the fraudulent product offered by firm L.

However, firm H does not always want to unshroud. In an SE, firm L and firm H both obtain some market power by offering differentiated products to two groups of investors. Both firms earn positive profits in this situation. However, reducing λ may change the market equilibrium into an NE. Recall that c_L^* is increasing in λ, so reducing λ has two effects: On the one hand, it raises the proportion of sophisticated investors, thereby making it more profitable to offer a normal product, and benefits firm H. On the other hand, as offering

[12]In markets of financial advisory, firms with clean records coexist with firms frequently engaging in misconduct (Egan et al., 2019). In the residential mortgage-backed securities market, there are firms that intentionally provide buyers false information on asset characteristics, which leads to severe losses to buyers (Piskorski et al., 2015).

a normal product becomes attractive, it may induce firm L to deviate from committing financial fraud and provide a normal product to compete with firm H. As a consequence, when δ is small, firm H may have to increase its rate of return offered to investors and forgo some of its profits; when δ is large, information disclosure may change an SE into an NE and drive firm H's profit down to zero. Therefore, whether firm H is willing to unshroud depends on the aggregation of these two opposite driving forces.

In the proof of Proposition 2, we show that firm H's profit in an SE is bounded above by firm L's incentive constraint. Thus, firm H's profit will be increased only when this incentive constraint still holds after information disclosure, otherwise unshrouding makes firm H worse off. Therefore, firm H will unshroud only if

$$(\lambda - \delta)\pi^1 - c_L \geq \pi^0 \;\Leftrightarrow\; \delta \leq \lambda - \frac{c_L + \pi^0}{\pi^1} \equiv \delta^*.$$

Here, we assume that firm H will unshroud if it is indifferent and that the market equilibrium will not switch in the boundary cases. Proposition 7 formally states this result.

Proposition 7. *There exists δ^* such that in an SE with $r_H = r^*$, firm H will unshroud if and only if $\delta \leq \delta^*$.*

In other words, when financial fraud is present, an honest firm has some private incentive to educate investors such that some naive investors who would be attracted by the fraudulent product become aware of the danger and purchase the normal product offered by the honest firm. However, the incentive is limited: The honest firm does not want to reduce the proportion of naive investors to the extent that offering a fraudulent product becomes unprofitable, which intensifies the competition in the normal product market.[13] From Proposition 7, δ^* decreases with c_L, implying that firm H is more likely to shroud if firm L's cost advantage in exploiting naive investors is small.

Proposition 7 provides a new perspective on the firm's incentive to shroud in a competitive market. Firm L has a cost advantage in exploiting naive investors and, thus, chooses not to compete with firm H for sophisticated investors. By shrouding, firm H restricts the profitability of offering a normal product and indirectly discourages firm L from competing with it.[14]

[13] If firms can control the proportion of investors receiving the disclosed information, i.e., δ becomes a choice variable, then firm H will choose $\delta = \delta^*$.

[14] In Appendix B, we consider an alternative model setting with simultaneous moves and show that the market segregation results preserve. However, Proposition 7 does not hold because firm H will always unshroud as it cannot affect firm L's decision.

4.3 Policy implications

Based on Section 4.2, we further analyze some regulatory issues in the leader-follower environment with information disclosure. We consider three policy instruments: increasing legal punishment, implementing a public education program, and lowering the interest rate ceiling. Increasing legal punishment raises firms' costs of committing financial fraud, while public education programs reduce firms' profits from offering fraudulent products. In the absence of private information disclosure, they are clearly beneficial to investors. Lowering the interest rate ceiling reduces the profitability of fraudulent products and, thus, improves investor welfare as long as it is not binding for normal products.

However, the welfare effects of these policies are ambiguous when competition and unshrouding are both present. All three policies may reduce firms' private incentive to disclose information. We focus on the SE case since it is most relevant to policy design.

Increasing legal punishment. Suppose that policymakers increase c_L to c_L'.[15] Such a policy can be interpreted as an increase in legal punishment, or auditing power, and thereby increases firm L's cost of committing financial fraud. As in the previous case, the direct effect of increasing c_L is to make financial fraud a less favorable choice for firm L, while the indirect effect is to make firm H less likely to disclose information about its choice set. Similarly, increasing c_L to c_L' will prevent firm H from unshrouding if and only if $\lambda - \frac{c_L'+\pi^0}{\pi^1} < \delta \leq \delta^*$.

If we further assume that $\lambda - \frac{c_L'+\pi^0}{\pi^1} \geq 0$, then firm H offers $r_H = r^*$ before and after the policy, and the policy does not transform the current SE into an NE. However, investor welfare is decreased because firm H loses its incentive to unshroud. Our analysis can be summarized in Proposition 8.

Proposition 8. *Suppose that $0 \leq \lambda - \frac{c_L'+\pi^0}{\pi^1} < \delta \leq \delta^*$. Then, raising c_L to c_L' is not welfare-improving.*

Implementing a public education program. Suppose that policymakers can also reduce the fraction of naive investors by implementing a public education program as in 4.1. The education program reduces λ to λ'. Then, according to Proposition 7, when $0 \leq \lambda' - \frac{c_L'+\pi^0}{\pi^1} < \delta \leq \delta^*$, the public education program does not change the rates of return offered by firm L and firm H but crowds out firm H's incentive to disclose information, which is effective in naive investors of measure δ. Therefore, if the public education program is not as effective as firm H's unshrouding behavior, i.e., $\lambda - \lambda' \leq \delta$, investor welfare will

[15] There is no need to increase c_H because firm H already offers a normal product in the SE.

not increase due to the crowding-out effect. Proposition 9 characterizes the condition under which the crowding-out effect occurs.

Proposition 9. *Suppose that $\lambda - \lambda' \leq \delta$ and that $0 \leq \lambda' - \frac{c_L' + \pi^0}{\pi^1} < \delta \leq \delta^*$. Then, reducing λ to λ' is not welfare-improving.*

Lowering the interest rate ceiling. Suppose that policymakers reduce $\overline{R}$ to $\overline{R}'$. Namely, a lower interest rate ceiling is imposed on the financial products sold in the market. Then, depending on the relationship between $\overline{R}'$ and r^*, there are two different cases.

If $\overline{R}' > r^*$ still holds, then the direct effect is to make the fraudulent product less attractive to naive investors, because firm L would advertise the highest possible rate of return for its product. Thus, financial fraud becomes less favorable to firm L, and fraudulent products are less likely to appear in the market. However, this policy also weakens firm H's incentive to unshroud because now firm L is more likely to deviate and become a competitor of firm H. Mathematically, a decrease in $\overline{R}$ implies an increase in $\overline{\alpha}(\overline{R})$, which leads to decreases in π^1 and δ^*. In particular, lowering $\overline{R}$ to $\overline{R}'$ will change firm H's decision on information disclosure from unshrouding to shrouding if

$$\lambda - \frac{c_L + \pi^0}{pRF(\overline{\alpha}(\overline{R}'))} < \delta \leq \delta^*, \tag{4}$$

where the left-hand side is the new cutoff that makes firm H indifferent between unshrouding and shrouding after the imposition of the policy.

We can also compute the welfare effect of lowering the interest rate ceiling. Suppose that (4) is satisfied. Before this policy, firm H will disclose, and thus, investors' welfare change is

$$\begin{aligned}\Delta W_{before} = (\lambda - \delta) &\int_0^{\overline{\alpha}(\overline{R})} \frac{(\omega - I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha) \\ &+ (1 - \lambda + \delta) \int_0^{\overline{\alpha}(r^*)} \frac{p(\omega + r^*)^{1-\alpha} + (1-p)(\omega - I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha).\end{aligned}$$

After this policy is implemented, firm H will shroud, and investor welfare is

$$\begin{aligned}\Delta W_{after} = \lambda &\int_0^{\overline{\alpha}(\overline{R}')} \frac{(\omega - I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha) \\ &+ (1 - \lambda) \int_0^{\overline{\alpha}(r^*)} \frac{p(\omega + r^*)^{1-\alpha} + (1-p)(\omega - I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha).\end{aligned}$$

Therefore, the welfare effect of this policy is negative if

$$\Delta W_{after} < \Delta W_{before}. \tag{5}$$

Proposition 10 summarizes our discussion.

Proposition 10. *Suppose that $\overline{R}' > r^*$ and that δ satisfies (4) and (5). Then, reducing $\overline{R}$ to $\overline{R}'$ is not welfare-improving.*

If $\overline{R}' \leq r^*$, the firm offering a normal product will find it optimal to choose $\overline{R}'$, since its profit function is nondecreasing in $[0, r^*]$. Similarly, the firm offering a fraudulent product will also choose $\overline{R}'$ to attract naive investors. No firm can deviate from offering $\overline{R}'$, but whether they will commit financial fraud depends on their costs. This policy is also distortionary and may be harmful to investors, since sophisticated investors cannot obtain a return of r^* from the honest firm.

In summary, the net effect of policy intervention is ambiguous when firms can disclose information to investors. If public policies cannot change an SE into an NE, or make the high-cost firm increase its rate of return offered to investors, the high-cost firm's incentive to unshroud may be weakened due to the fear of inducing competitors. Based on similar logic, a public education program may crowd out firms' unshrouding behaviors.

5 Conclusion

Widespread financial fraud has emerged as a pressing problem in many countries. Investors are attracted by unrealistically high returns because some of them may not have a proper awareness of the underlying high risks and the possibility of financial fraud. In this paper, we model how boundedly rational investors get exploited by firms providing fraudulent financial products. Educating investors to reduce the proportion of naive investors not only directly helps these investors but also attenuates or even eliminates the firms' incentive to commit financial fraud, which is beneficial to the development of a healthy financial market.

Our model offers several novel implications on competition and public policies. Having naive investors in the market, there exists a separating equilibrium in which two firms offer differentiated products targeting two types of investors. Introducing a new firm to the market may lower investor welfare because naive investors will purchase the fraudulent product that is not offered in the monopoly market. In this separating equilibrium, the honest firm offering a normal product to sophisticated investors may have an incentive to unshroud the possibility of financial fraud because doing so increases its market share. However, if the proportion of naive investors becomes too small, the previously dishonest firm will turn to compete for sophisticated investors. Because of the second force, the honest firm may be reluctant to take the welfare-improving action of disclosing information. Lowering the interest rate ceiling, increasing legal punishment, and implementing a public education program may all discourage the honest firm from unshrouding. Therefore, in protecting boundedly rational consumers, policymakers should devote more attention to the key mechanisms behind nudges to avoid undesirable outcomes.

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Appendix

A An Alternative Representation of the Model

When players have different awareness, the standard notion of extensive-form games cannot be applied as players may have different views on the structure of the game. In a recent study, Heifetz et al. (2013) define a new concept *generalized extensive-form game* which allows for unaware players. In this section, we show that our model can also be interpreted as a generalized extensive-form game.

Consider our benchmark model in which a firm sells financial products to a number of investors. First, the firm chooses whether to commit financial fraud, which is denoted by $x \in \{0, 1\}$. Then, the firm posts a rate of return r for its product. Investors observe r and decide whether they accept the firm's offer. For sophisticated investors, the game tree is depicted in the left part of Figure A.1. For naive investors, the game tree is depicted in the right part of Figure A.1. As naive investors are unaware of the firm's choice of committing financial fraud, their understanding of the game structure is incomplete. Graphically, in their perceived game tree the firm's first decision node is missing. Information disclosure changes naive investors into sophisticated ones, therefore changes their perceived game tree from the right part to the left part of Figure A.1.

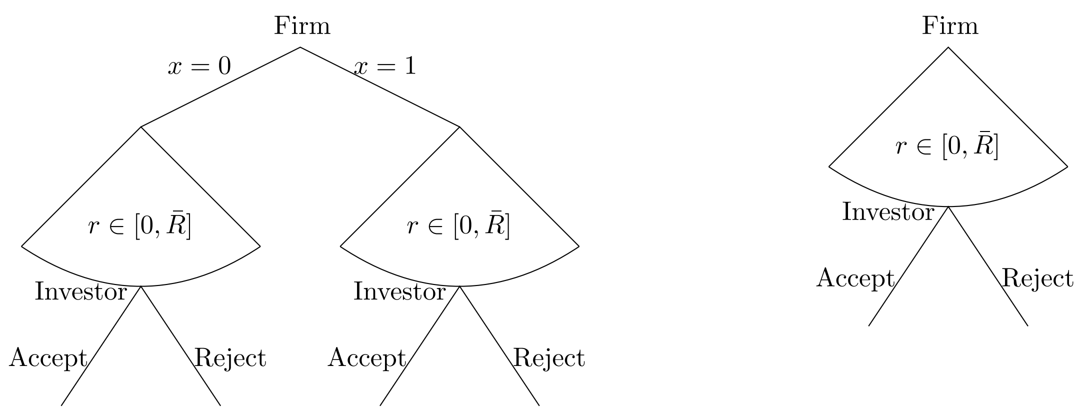

Figure A.1: The generalized extensive-form game

B Simultaneous Moves

In Section 3, we consider a leader-follower game and assume that two firms move sequentially. Now we study a variation of our model in which two firms move simultaneously. It can be shown that the market segmentation results preserve in this alternative setting.

Normal-product equilibrium (NE). In an NE, both firms offer normal products ($x_H = x_L = 0$). Competition drives returns up to $r_H = r_L = R$, and both firms earn zero profit. If firm j deviates to choosing $x_j = 1$, the most profitable strategy is to set $r_j = \overline{R}$. Naive investors will purchase only from firm j, and sophisticated investors will purchase only from the other firm, denoted firm k ($k \neq j$). Therefore, by choosing $x_j = 1$, firm j's profit is $\lambda\pi^1 - c_j$. Hence, $x_H = x_L = 0$ constitutes an equilibrium if,

$$\lambda\pi^1 - c_j \leq 0 \;\Leftrightarrow\; c_j \geq \lambda\pi^1 \equiv c_L^* \text{ for } j = L, H.$$

Because $c_L \leq c_H$, an NE with $r_H = r_L = R$ arises when $c_L \geq c_L^*$.

Fraudulent-product equilibrium (FE). In an FE, both firms commit financial fraud ($x_H = x_L = 1$). They will propose $r_H = r_L = \overline{R}$ to attract as many naive investors as possible, so each firm earns a profit $\lambda\pi^1/2 - c_j$.

If firm j deviates to $x_j = 0$, then the most profitable rate of return is $r_j = r^*$, which yields a profit $(1-\lambda)\pi^0$. $x_H = x_L = 1$ constitutes an equilibrium if

$$\frac{1}{2}\lambda\pi^1 - c_j \geq (1-\lambda)\pi^0 \;\Leftrightarrow\; c_j \leq \frac{1}{2}\lambda\pi^1 - (1-\lambda)\pi^0 \equiv c_H^*, \text{ for } j = L, H.$$

Because $c_L \leq c_H$, an FE with $r_H = r_L = \overline{R}$ arises when $c_H \leq c_H^*$. Note that $c_H^* \geq 0$ requires $\lambda \geq \overline{\lambda}$. Hence, when the proportion of naive investors is sufficiently small, c_H^* becomes negative, implying that there is no FE.

Separating equilibrium (SE). Suppose that, in an SE, firm j provides a normal product while firm k commits financial fraud, i.e. $x_j = 0$, $x_k = 1$.

In an SE, firm j's equilibrium strategy is $(r^*, 0)$, and firm k's equilibrium strategy is $(\overline{R}, 1)$. For firm j playing $x_j = 0$, the most profitable deviation from offering a normal product is to play $(\overline{R}, 1)$. Therefore, its incentive constraint is

$$(1-\lambda)\pi^0 \geq \frac{1}{2}\lambda\pi^1 - c_j \;\Leftrightarrow\; c_j \geq \frac{1}{2}\lambda\pi^1 - (1-\lambda)\pi^0 = c_H^*.$$

For firm k playing $x_k = 1$, its most profitable deviation is to play $(r^* + \varepsilon, 0)$, i.e., to offer a normal product with $r_k = r^* + \varepsilon$, where ε is a small positive number. Upon deviating, firm k is able to capture the whole market and earn a profit arbitrarily close to π^0. Therefore, firm k's incentive constraint is

$$\lambda\pi^1 - c_k \geq \pi^0 \;\Leftrightarrow\; c_k \leq \lambda\pi^1 - \pi^0 = c_L^* - \pi^0.$$

Hence, an SE can be supported if $c_j \geq c_H^*$ and $c_k \leq c_L^* - \pi^0$. These two conditions are equivalent to $c_H \geq c_H^*$ and $c_L \leq c_L^* - \pi^0$.

Proposition 11 summarizes our results when two firms move simultaneously.

Proposition 11. *There exist c_H^* and c_L^* such that:*

(a). When $c_L \geq c_L^*$, there exists an NE in which both firms play $(R, 0)$.

(b). When $c_H \leq c_H^*$, there exists an FE in which both firms play $(\overline{R}, 1)$.

(c). When $c_H \geq c_H^*$ and $c_L \leq c_L^* - \pi^0$, there exists an SE in which firm H plays $(r^*, 0)$, and firm L plays $(\overline{R}, 1)$.

Therefore, our market segregation results preserve in this simultaneous-move setting.

C Proofs

C.1 Proof of Proposition 2

Normal-product equilibrium (NE). In an NE, both firms offer normal products ($x_H = x_L = 0$). Suppose that firm H chooses $r_H < R$, then firm L's best response would be choosing r_L slightly higher than r_H to capture the whole market. Therefore, $r_H < R$ cannot be played in an NE. Given that firm H chooses $r_H = R$ in the equilibrium, firm L's equilibrium strategy must be $r_L = R$, and both firms' equilibrium profit is zero.

If firm L deviates to choosing $x_L = 1$, the most profitable strategy is setting $r_L = \overline{R}$ to attract as many naive investors as possible. Upon such deviation, firm L's profit is $\lambda\pi^1 - c_L$. Thus, firm L's incentive constraint is

$$\lambda\pi^1 - c_L \leq 0 \;\Leftrightarrow\; c_L \geq \lambda\pi^1 \equiv c_L^*.$$

Now suppose $c_L \geq c_L^*$. If firm H deviates to choosing $x_H = 1$, it will also set $r_H = \overline{R}$. Observing firm H's deviation, firm L will stick to $x_L = 0$. Firm H's profit upon deviation is $\lambda\pi^1 - c_H$, which is nonpositive. Hence, an NE exists if and only if $c_L \geq c_L^*$.

Fraudulent-product equilibrium (FE). In an FE, both firms commit financial fraud ($x_H = x_L = 1$). They will propose $r_H = r_L = \overline{R}$ to attract as many naive investors as possible, so each firm earns a profit $\lambda\pi^1/2 - c_j$.

If firm L deviates to choosing $x_L = 0$, the most profitable strategy is setting $r_L = r^*$ to maximize its profit from selling a normal product to sophisticated investors. Upon such deviation, firm L's profit is $(1-\lambda)\pi^0$. Thus, firm L's incentive constraint is

$$\frac{1}{2}\lambda\pi^1 - c_L \geq (1-\lambda)\pi^0 \;\Leftrightarrow\; c_L \leq \frac{1}{2}\lambda\pi^1 - (1-\lambda)\pi^0 \equiv c_H^*.$$

If firm H deviates to choosing $x_H = 0$ and $r_H < R$, it can make a positive profit only when firm L does not follow its deviation and sticks to $x_L = 1$. Put differently, if firm L also deviates and plays $x_L = 0$, it will capture the whole market by setting r_L slightly above r_H. Firm H only gets zero profit from deviation. This is the case if

$$\lambda\pi^1 - c_L \leq p(R - r_H)F(\overline{\alpha}(r_H)). \tag{6}$$

Moreover, even if (6) is violated, and firm H can make a positive profit upon deviation, it is also possible that such profit is less than firm H's equilibrium profit. In other words, firm H's incentive constraint is

$$\frac{1}{2}\lambda\pi^1 - c_H \geq (1-\lambda)p(R - r_H)F(\overline{\alpha}(r_H)). \tag{7}$$

Combining (6) and (7) gives us either $c_H \leq c_H^*$, or

$$c_H - (1-\lambda)c_L \leq (\lambda - \frac{1}{2})\lambda\pi^1.$$

Hence, an FE exists if and only if $c_L \leq c_H^*$ and $c_H \leq c_H^* + (1-\lambda)\max\{0, c_L - \lambda\pi^1 + \pi^0\}$.

Separating equilibrium (SE). Suppose that, in an SE, firm H provides a normal product with $r_H < R$, while firm L commits financial fraud with $r_L = \overline{R}$. Firm H's equilibrium profit is $(1-\lambda)p(R - r_H)F(\overline{\alpha}(r_H))$, and firm L's equilibrium profit is $\lambda\pi^1 - c_L$.

If firm L deviates to choosing $x_L = 0$, its most profitable strategy is setting r_L slightly above r_H to capture the whole market. Thus, firm L's incentive constraint is

$$\lambda\pi^1 - c_L \geq p(R - r_H)F(\overline{\alpha}(r_H)).$$

A necessary condition is $c_L \leq c_L^*$. Note that, if the right-hand side is strictly less than π^0, this constraint must be binding, otherwise firm H will change its r_H to increase its profit. In other words,

$$p(R - r_H)F(\overline{\alpha}(r_H)) = \min\{\lambda\pi^1 - c_L, \pi^0\}.$$

If firm H deviates to choosing $x_H = 1$ and $r_H = \overline{R}$, firm L will either sticks to $x_L = 1$ or switches to $x_L = 0$. The former case happens when

$$\frac{1}{2}\lambda\pi^1 - c_L \geq (1-\lambda)\pi^0 \;\Leftrightarrow\; c_L \leq c_H^*.$$

In this case, firm H will earn $\frac{1}{2}\lambda\pi^1 - c_H$ upon deviation. Its incentive constraint is

$$(1-\lambda)p(R-r_H)F(\overline{\alpha}(r_H)) \geq \frac{1}{2}\lambda\pi^1 - c_H,$$

which implies $c_H \geq c_H^* + (1-\lambda)\max\{0, c_L - \lambda\pi^1 + \pi^0\}$. The latter case happens when

$$\frac{1}{2}\lambda\pi^1 - c_L \leq (1-\lambda)\pi^0 \;\Leftrightarrow\; c_L \geq c_H^*.$$

In this case, firm H will earn $\lambda\pi^1 - c_H$ upon deviation. Its incentive constraint is

$$(1-\lambda)p(R-r_H)F(\overline{\alpha}(r_H)) \geq \lambda\pi^1 - c_H,$$

which implies $c_H \geq c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$.

In particular, there exists an SE with $r_H = r^*$ if

$$\begin{aligned}
&c_L \leq \lambda\pi^1 - \pi^0,\\
&c_H \geq c_H^* \text{ if } c_L \leq c_H^*,\\
&c_H \geq c_H^* + \frac{1}{2}\lambda\pi^1 \text{ if } c_L \geq c_H^*.
\end{aligned}$$

Consider an alternative case where firm H commits financial fraud with $r_H = \overline{R}$, and firm L provides a normal product with $r_L = r^*$ in an SE. By a similar discussion, firm L's incentive constraint is

$$(1-\lambda)\pi^0 \geq \frac{1}{2}\lambda\pi^1 - c_L \;\Leftrightarrow\; c_L \geq c_H^*.$$

If firm H deviates to choosing $x_H = 0$, firm L will stick to $x_L = 0$ when

$$p(R-r_H)F(\overline{\alpha}(r_H)) \geq \lambda\pi^1 - c_L. \tag{8}$$

In this case, firm H's profit upon deviation is zero. Alternatively, firm L will switch to $x_L = 1$ when (8) is violated. In this case, firm H's incentive constraint is

$$\lambda\pi^1 - c_H \geq (1-\lambda)p(R-r_H)F(\overline{\alpha}(r_H)). \tag{9}$$

Combining (8) and (9) gives us $c_H \leq c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$.

In sum, an SE exists if and only if $c_L \geq c_H^*$ or $c_H \geq c_H^* + (1-\lambda)\max\{0, c_L - \lambda\pi^1 + \pi^0\}$.

C.2 Proof of Proposition 3

Proving part (a) of the proposition is straightforward, so we only need to verify part (b).

In an NE, investors with $\alpha \leq \overline{\alpha}(R)$ purchase both firms' normal products with equal probability. Investors' welfare change is

$$\Delta W_{NE}^c = \int_0^{\overline{\alpha}(R)} \frac{p(\omega+R)^{1-\alpha} + (1-p)(\omega-I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha).$$

Compared to the welfare with a monopoly offering a normal product given in (2), we find that $\Delta W_{NE}^c > \Delta W_N^m$, i.e., competition improves welfare because the rate of return for the normal product increases from r^* to R.

In an FE, naive investors with $\alpha \leq \overline{\alpha}(\overline{R})$ purchase both firms' fraudulent products with equal probability. Investors' welfare change is

$$\Delta W_{FE}^c = \lambda \int_0^{\overline{\alpha}(\overline{R})} \frac{(\omega-I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha),$$

Compared to the welfare with a monopoly offering a fraudulent product given in (3), we find that $\Delta W_{FE}^c = \Delta W_F^m$, i.e., competition does not improve welfare if firms offer fraudulent products.

In an SE with the normal product's rate of return being r^*, sophisticated investors with $\alpha \leq \overline{\alpha}(r^*)$ purchase firm H's normal product, and naive investors with $\alpha \leq \overline{\alpha}(\overline{R})$ purchase firm L's fraudulent product. Investors' welfare change is

$$\begin{aligned}\Delta W_{SE}^c = {} & \lambda \int_0^{\overline{\alpha}(\overline{R})} \frac{(\omega-I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha) \\ & + (1-\lambda) \int_0^{\overline{\alpha}(r^*)} \frac{p(\omega+r^*)^{1-\alpha} + (1-p)(\omega-I)^{1-\alpha} - \omega^{1-\alpha}}{1-\alpha} dF(\alpha),\end{aligned}$$

which lies in between welfare measures ΔW_N^m and ΔW_F^m.

C.3 Proof of Proposition 4

We consider the following cases separately:

- Case 1: Firm H offers a normal product; firm L enters and offers a fraudulent product;

- Case 2: Firm H offers a normal product; firm L does not enter;
- Case 3: Firm H offers a fraudulent product; firm L enters and offers a normal product;
- Case 4: Firm H offers a fraudulent product; firm L enters and offers a fraudulent product;
- Case 5: Firm H offers a fraudulent product; firm L does not enter.

Case 1. Firm L's incentive constraint is

$$\lambda\pi^1 - c_L \geq \max\{\kappa, p(R - r_H)F(\overline{\alpha}(r_H))\}. \tag{10}$$

If firm H deviates to offering a fraudulent product, firm L's profit upon entry is $\max\{\lambda\pi^1/2 - c_L, (1-\lambda)\pi^0\} - \kappa$. Thus, there are three sub-cases.

If $c_L \leq c_H^*$ and $\kappa \leq \lambda\pi^1/2 - c_L$, then firm L enters and offers a fraudulent product. Firm H's incentive constraint is

$$(1-\lambda)p(R - r_H)F(\overline{\alpha}(r_H)) \geq \frac{1}{2}\lambda\pi^1 - c_H. \tag{11}$$

By (10) and (11), an equilibrium exists if $c_H \geq c_H^*$.

If $c_L \geq c_H^*$ and $\kappa \leq (1-\lambda)\pi^0$, then firm L enters and offers a normal product. If $\kappa \geq \max\{\lambda\pi^1/2 - c_L, (1-\lambda)\pi^0\}$, then firm L does not enter. In both cases, firm H's incentive constraint is

$$(1-\lambda)p(R - r_H)F(\overline{\alpha}(r_H)) \geq \lambda\pi^1 - c_H. \tag{12}$$

By (10) and (12), an equilibrium exists if $c_L \leq c_L^* - \kappa$ and $c_H \geq c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$.

Case 2. Firm L's incentive constraint is

$$\kappa > \max\{p(R - r_H)F(\overline{\alpha}(r_H)), \lambda\pi^1 - c_L\}. \tag{13}$$

When firm H deviates to offering a fraudulent product, firm L has no incentive to enter and commit financial fraud because of (13). It will either offer a normal product, or simply do not enter. In both cases, firm H's incentive constraint is

$$p(R - r_H)F(\overline{\alpha}(r_H)) \geq \lambda\pi^1 - c_H. \tag{14}$$

By (13) and (14), an equilibrium exists if $\kappa \geq c_L^* - c_L$ and $c_H \geq \lambda\pi^1 - \pi^0$.

Case 3. Firm L's incentive constraint is

$$\kappa \le (1-\lambda)\pi^0,$$
$$c_L \ge c_H^*.$$

Suppose that firm H deviates to offering a normal product with a rate of return r_H. When

$$p(R-r_H)F(\overline{\alpha}(r_H)) \ge \max\{\kappa, \lambda\pi^1 - c_L\}, \tag{15}$$

firm L will follow by offering a normal product with a rate of return slightly higher than r_H.

When (15) is violated and $\lambda\pi^1 - c_L \ge \kappa$, firm L will follow by offering a fraudulent product. Firm H's incentive constraint is

$$\lambda\pi^1 - c_H \ge (1-\lambda)p(R-r_H)F(\overline{\alpha}(r_H)). \tag{16}$$

Any $r_H \in [r^*, R]$ should satisfy either (15) or (16). Therefore, an equilibrium exists if $c_H \le c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$.

When (15) is violated and $\lambda\pi^1 - c_L < \kappa$, firm L will not enter. Firm H's incentive constraint is

$$\lambda\pi^1 - c_H \ge p(R-r_H)F(\overline{\alpha}(r_H)). \tag{17}$$

Similarly, an equilibrium exists if $c_H \le \lambda\pi^1 - \pi^0$.

Case 4. Firm L's incentive constraint is

$$\kappa \le \frac{1}{2}\lambda\pi^1 - c_L,$$
$$c_L \le c_H^*.$$

The analysis of firm H's incentives directly follows Case 3. Suppose that firm H deviates to offering a normal product with a rate of return r_H. When (15) is satisfied, firm L will follow by offering a normal product with a rate of return slightly higher than r_H.

When (15) is violated, since $\lambda\pi^1 - c_L > \kappa$, firm L will follow by offering a fraudulent product. Firm H's incentive constraint is

$$\frac{1}{2}\lambda\pi^1 - c_H \ge (1-\lambda)p(R-r_H)F(\overline{\alpha}(r_H)). \tag{18}$$

Therefore, an equilibrium exists if $c_H \le c_H^*$.

Case 5. Firm L's incentive constraint is

$$\kappa > \max\{(1-\lambda)\pi^0, \frac{1}{2}\lambda\pi^1 - c_L\}.$$

Again, the analysis of firm H's incentives follows Case 3. Therefore, an equilibrium exists if $c_L \le c_L^* - \kappa$ and $c_H \le c_L^* - (1-\lambda)\min\{\lambda\pi^1 - c_L, \pi^0\}$, or $c_L \ge c_L^* - \kappa$ and $c_H \le \lambda\pi^1 - \pi^0$.

Summarizing all five cases, we have the results stated in the proposition.

C.4 Proof of Proposition 5

Case 1: $q_L = q_F = 0$. In this case, all firms commit financial fraud and propose a rate of return $\overline{R}$ to attract as many naive investors as possible. For any $j \in \{1, \dots, m\}$, firm j's profit is $\lambda\pi^1/m - c_j$. This profit should be non-negative, implying that

$$\frac{1}{m}\lambda\pi^1 - c_1 \ge 0.$$

If one of the followers, say firm $k+1$, deviates to choosing $x_{k+1} = 0$, its most profitable strategy is setting $r_{k+1} = r^*$ to maximize its profit from selling a normal product to sophisticated investors. Upon such deviation, firm $k+1$'s profit is $(1-\lambda)\pi^0$. Thus, firm $k+1$'s incentive constraint is

$$\frac{1}{m}\lambda\pi^1 - c_{k+1} \ge (1-\lambda)\pi^0,$$

which implies

$$c_{k+1} \le \frac{1}{m}\lambda\pi^1 - (1-\lambda)\pi^0 = \underline{c}. \tag{19}$$

If one of the leaders, say firm 1, deviates to choosing $x_1 = 0$ and $r_1 < R$, it can make a positive profit only when all followers do not follow its deviation and stick to committing financial fraud. Put differently, if any of the followers, say firm j ($j \ge k+1$) also deviates and offers a normal product, it will capture the whole market by setting a rate of return slightly above r_1. Firm 1 thus only gets zero profit from deviation. This is the case if

$$\frac{1}{m-1}\lambda\pi^1 - c_j \le p(R - r_1)F(\overline{\alpha}(r_1)). \tag{20}$$

Moreover, even if (20) is violated, and the leader can make a positive profit upon deviation, it is also possible that such profit is less than its equilibrium profit. In other words, firm 1's incentive constraint is

$$\frac{1}{m}\lambda\pi^1 - c_1 \geq (1-\lambda)p(R-r_1)F(\overline{\alpha}(r_1)). \tag{21}$$

Any r_1 should satisfy either (20) or (21). Note that by (19), the left-hand side of (20) must be positive. This implies either

$$\frac{1}{m}\lambda\pi^1 - c_1 \geq (1-\lambda)\pi^0,$$

or

$$\frac{1}{m}\lambda\pi^1 - c_1 \geq (1-\lambda)[\frac{1}{m-1}\lambda\pi^1 - c_j].$$

The first inequality gives us

$$c_1 \leq \underline{c}, \tag{22}$$

and the second inequality gives us

$$c_1 - (1-\lambda)c_j \leq (\frac{1}{m} - \frac{1-\lambda}{m-1})\lambda\pi^1. \tag{23}$$

It is without loss of generality to let $j = k+1$ in (23). Therefore, an equilibrium exists if

$$\begin{aligned} c_1 &\leq \underline{c} + (1-\lambda)\max\{0, c_{k+1} - \frac{\overline{c}}{m-1} + \pi^0\}, \\ c_{k+1} &\leq \underline{c}. \end{aligned}$$

Case 2: $q_L = 1$, $q_F = 0$. In this case, only firm 1 offers normal products. Suppose that firm 1 posts a rate of return r_1. Then, its profit is $(1-\lambda)p(R-r_1)F(\overline{\alpha}(r_1))$. If firm 1 deviates to committing financial fraud, either all followers stick to committing financial fraud, or one of the followers switches to normal products. The former case happens if

$$\frac{1}{m}\lambda\pi^1 - c_{k+1} \geq (1-\lambda)\pi^0,$$

which is equivalent to $c_{k+1} \leq \underline{c}$. In this case, firm 1's incentive constraint is

$$(1-\lambda)p(R-r_1)F(\overline{\alpha}(r_1)) \geq \frac{1}{m}\lambda\pi^1 - c_1. \tag{24}$$

In the latter case, $c_{k+1} > \underline{c}$, firm 1's incentive constraint is

$$(1-\lambda)p(R-r_1)F(\overline{\alpha}(r_1)) \geq \frac{1}{m-1}\lambda\pi^1 - c_1, \tag{25}$$

Moreover, in both cases, other firms should have no incentive to offer normal products, implying that

$$\frac{1}{m-1}\lambda\pi^1 - c_2 \geq (1-\lambda)p(R-r_1)F(\overline{\alpha}(r_1)). \tag{26}$$

Therefore, when $c_{k+1} \leq \underline{c}$, an equilibrium exists if (24) and (26) hold for some r_1, which is equivalent to $c_1 \geq \underline{c}$ and $c_2 \leq \overline{c}/(m-1)$.

When $c_{k+1} > \underline{c}$, an equilibrium exists if (25) and (26) hold for some r_1, which can be simplified as $c_1 \geq \overline{c}/(m-1) - (1-\lambda)\pi^0$ and $c_2 \leq \overline{c}/(m-1)$.

Case 3: $q_L = 0$, $q_F = 1$. In this case, only firm $k+1$ offers normal products. Suppose that firm $k+1$ posts a rate of return r_{k+1}. Then, its incentive constraint is

$$(1-\lambda)p(R-r_{k+1})F(\overline{\alpha}(r_{k+1})) \geq \frac{1}{m}\lambda\pi^1 - c_{k+1}. \tag{27}$$

If one of the leaders, say firm 1, deviates to offering normal products with a rate of return r_1, either firm $k+1$ sticks to offering normal products with a rate of return slightly above r_1, or firm $k+1$ switches to committing financial fraud. Consider the following inequality

$$(1-\lambda)p(R-r_1)F(\overline{\alpha}(r_1)) \geq \frac{1}{m-1}\lambda\pi^1 - c_{k+1}. \tag{28}$$

If (28) holds for any r_1, then firm 1's profit upon deviation is zero, implying that it should have no incentive to deviate. If (28) holds for some (but not all) r_1, firm 1's profit upon deviation should be capped by the right-hand side of (28). Such deviation is attractive for firm 1 because the right-hand side of (28) is no smaller than firm 1's equilibrium profit. If (28) never holds, firm 1 has no incentive to deviate if

$$\frac{1}{m-1}\lambda\pi^1 - c_1 \geq (1-\lambda)\pi^0.$$

Therefore, firm 1's incentive constraint can be summarized as

(a). $c_{k+1} \geq \overline{c}/(m-1)$; or

(b). $c_1 \leq \overline{c}/(m-1) - (1-\lambda)\pi^0$.

Moreover, other followers should have no incentive to deviate to offering normal products, which implies

$$(1-\lambda)p(R-r_{k+1})F(\overline{\alpha}(r_{k+1})) \leq \frac{1}{m-1}\lambda\pi^1 - c_{k+2}. \tag{29}$$

(27) and (29) should hold jointly for some r_{k+1}. They are equivalent to the following inequalities:

$$\frac{1}{m}\lambda\pi^1 - c_{k+1} \leq (1-\lambda)\pi^0,$$
$$\frac{1}{m-1}\lambda\pi^1 - c_{k+2} \geq 0.$$

Also note that we need $c_{k+1} \geq \overline{c}/(m-1)$. Therefore, an equilibrium with $q_L = 0$, $q_F = 1$ exists if $c_{k+1} \geq \underline{c}$ and $c_{k+2} \leq \overline{c}/(m-1) \leq c_{k+1}$.

Case 4: $1 < q_L + q_F = q < m$**.** In this case, firms $1,...,q_L$ (when $q_L > 0$) and $k+1, ..., k+q_F$ (when $q_F > 0$) offer normal products, and price competition will drive their profits down to zero. Hence, all these firms set their rates of return at R. The incentive constraint for followers offering normal products should be

$$0 \geq \frac{1}{m-q+1}\lambda\pi^1 - c_{k+q_F}.$$

If one of the leaders, say firm q_L, deviates to committing financial fraud, either all followers stick to their equilibrium strategies, or one of the followers, say firm $k+1+q_F$, switches to normal products. The former case happens if

$$\frac{1}{m-q+1}\lambda\pi^1 - c_{k+1+q_F} \geq 0,$$

which is equivalent to $c_{k+1+q_F} \leq \overline{c}/(m-q+1)$. In this case, firm q_L's incentive constraint is

$$0 \geq \frac{1}{m-q+1}\lambda\pi^1 - c_{q_L}. \tag{30}$$

In the latter case, $c_{k+1+q_F} > \overline{c}/(m-q+1)$, firm q_L's incentive constraint is

$$0 \geq \frac{1}{m-q}\lambda\pi^1 - c_{q_L}. \tag{31}$$

Moreover, in both cases, other firms should have no incentive to offer normal products, implying that

$$\frac{1}{m-q}\lambda\pi^1 - c_{q_L+1} \geq 0. \tag{32}$$

Therefore, when $(m-q+1)c_{k+q_F} \geq \bar{c} \geq (m-q+1)c_{k+1+q_F}$, an equilibrium exists if

$$(m-q+1)c_{q_L} \geq \bar{c} \geq (m-q)c_{q_L+1}.$$

When $\bar{c} < (m-q+1)c_{k+1+q_F}$, an equilibrium exists if

$$(m-q)c_{q_L} \geq \bar{c} \geq (m-q)c_{q_L+1}.$$

Case 5: $q_L + q_F = m$. In this case, all firms offer normal products and earn zero profit. If one of the followers, say firm j ($j \geq k+1$), deviates to choosing $x_j = 1$, its most profitable strategy is setting $r_j = \overline{R}$ to attract as many naive investors as possible. Upon such deviation, firm j's profit is $\lambda\pi^1 - c_j$. Thus, firm j's incentive constraint is

$$\lambda\pi^1 - c_j \leq 0,$$

which implies

$$c_j \geq \lambda\pi^1 = \bar{c}. \tag{33}$$

(33) holds for all $j \geq k+1$ is equivalent to $c_m \geq \bar{c}$.

Now suppose $c_m \geq \bar{c}$. If one of the leaders, say firm j ($j \leq k$) deviates to choosing $x_j = 1$, it will set $r_j = \overline{R}$. Observing the leader's deviation, followers will stick to producing normal products because $1/2 \cdot \lambda\pi^1 - c_m \leq 0$. The leader's profit upon deviation is $\lambda\pi^1 - c_j$, which is nonpositive. Hence, an equilibrium exists if and only if $c_m \geq \bar{c}$.

C.5 Proof of Proposition 6

Part (a). In a monopoly, the firm always sells a normal product at the rate of return r^* since it is costless. The firm sells an additional fraudulent product if

$$\lambda\pi^1 + (1-\lambda)\pi^0 - c \geq \pi^0 \;\Leftrightarrow\; c \leq \lambda(\pi^1 - \pi^0).$$

Part (b). Similar to the monopoly case, in a duopoly, each firm always sells a normal product at the rate of return R. Moreover, if the leader offers an additional fraudulent product in the equilibrium, the follower will offer the same product for sure since $c_H \geq c_L$. Hence, the leader offers a fraudulent product if $c_H \leq \lambda\pi^1/2$. The follower offers a fraudulent product if $c_L \leq \lambda\pi^1/2$, or $\lambda\pi^1/2 < c_L \leq \lambda\pi^1$ and $c_H > \lambda\pi^1/2$.