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Keywords: Relational Contracts, Centralization, Hierarchy, Supplier Networks

JEL Classification: D23, D82, D86

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Relational Contracts and Hierarchy*

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May 2, 2022

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1 Introduction

Prominent management thinkers such as Peter Drucker and Henry Mintzberg likened the role of management to that of a maestro in an orchestra (Mintzberg, 1998, p. 141; Drucker, 2007, p. 77). Peter Drucker went on further predicting in 1989 that future businesses would be modeled on a symphony like Mahler's Eighth, where a single conductor leads more than 1000 participants without any intermediaries or assistants (Leavitt, 2003). Despite this prediction, hierarchies and organizational pyramids thrive. Many real-world organizations are structured as *multi-tier hierarchies* in which authorities are delegated through a vertical chain of command, rather than centralization where a single party has span of control that covers the entire organization.

What are the advantages of multi-tier hierarchies over centralization? When it is possible to write complete contracts contingent on all verifiable information, the revelation principle tells us that centralization cannot be dominated by any other organizational structures. Thus, a necessary condition for the prevalence of hierarchy is that some of the key variables that affect organizational surplus are not verifiable and hence non-contractible, as is often the case in reality. Another salient feature of real-world organizations is that many transactions occur repeatedly over the long term. Examples include supply relationships between firms or on-going employment relationships. Given the non-verifiability of information and repeated transactions, trading parties need to commit to informally agreed-upon promises in a way that is self-enforcing. These informal agreements are often enforced by so-called *relational contracts* based on long-term relationships (Malcomson, 2013).

The purpose of this paper to study relational contracts among multiple players and identify conditions under which the optimal allocation of contracting authorities is hierarchical or centralized. By doing so, we generate new insight into when cooperative long-term relationships should be governed by multi-tier hierarchy. As a result, we provide a new theory of hierarchy in a relational contracting environment under which multi-tier hierarchy can outperform centralization. To the best of our knowledge, this has not been shown in the existing literature.

We sketch our mode below, describe our main results, and provide the intuition behind them. There are three players, a principal (she) and agents 1 and 2 (he), who interact with one another repeatedly over time. In each period, each agent privately makes a binary effort choice - high or low -, and their joint efforts yield a stochastic output for the principal. The realized output can be observed by all players but is not verifiable by outside parties, implying that no formal output-based contracts can be written. Instead, players must rely on relational contracts in each period which comprise up-front transfers and ex-post payments, called informal bonuses, that are informally agreed upon. We compare two organizational structures that govern contracting relationships. Under *centralization*, the principal directly contracts with both agents. Under *hierarchy*, the principal contracts with only one agent, say agent 1, who is delegated the authority to contract with agent 2. This creates a three-tier hierarchy with agent 1 at the middle tier and agent 2 at the bottom tier.

Our primary aim is to characterize the necessary and sufficient conditions for implementing the first best under each organizational structure, when we call the organizational structure optimal. We define the first best as the outcome achievable if all the relevant information is verifiable. By our assumptions, the first best involves both agents choosing high effort in each period. Since our focus is on the equilibrium that implements the first best, equilibrium contracts should necessarily provide incentives to each agent to choose high effort. Then the central issue boils down to how to control deviation incentives by the players with contracting authority, where the deviation means refusal to pay the informal bonus to the counterparty in the contract.

Following Andrews and Barron (2016), we assume that payments are observed only by the two relevant parties, and so is any deviation from making the informally agreed-upon bonus payment. This assumption of *secret* deviation can be deemed reasonable when communication among players is limited. It also makes the question of organization design meaningful in our model. If the payment history is publicly observed by all three players, then we can show that centralization and hierarchy are equivalent in the sense that both organizational structures have the same necessary and sufficient conditions for the first best. Given the assumption of secret deviation, we can think of a key difference between centralization and hierarchy as how information is allocated among different players, which in turn affects their deviation incentives.

Under centralization, the principal contracts with both agents and, therefore, is the only player with full information. This allows her to make a secret deviation against one of the agents without triggering the punishment from the other agent. When the principal secretly deviates against only one agent, multilateral punishment is not possible, implying that centralization is susceptible to the principal's deviation. Thus, centralization is likely to achieve the first best in an environment where the principal's deviation incentives are weak. Under hierarchy, agent 1 as the agent at the middle tier is the only player with full information, who can make a secret deviation against agent 2, which can trigger the punishment from agent 2. However, because the principal cannot observe agent 1's deviation against agent 2, she cannot directly punish agent 1. Instead, the principal can rely on indirect punishment by choosing the informal bonus that can be foregone with a higher probability when agent 1 deviates against agent 2. This implies that the principal has to choose a larger informal bonus for agent 1 than under centralization. On the other hand, the principal's deviation incentives are weakened under hierarchy. It is because her deviation against agent 1 can instigate agent 1's deviation against agent 2. That is, hierarchy can enable multilateral punishment. Consequently, hierarchy is less prone to the principal's deviation than centralization, and is likely to achieve the first best when agents have less incentives to deviate.

Based on the preceding discussions, we can characterize the conditions for the first best using two key parameters. The first parameter is the common discount factor players use to discount future payoffs. We interpret the discount factor to reflect the level of trust players have on each other that the relationship will be long lasting and stable. Then our previous discussions tell us that trust is especially important in achieving the first best under hierarchy. It is because the punishment for deviation by the agent at the middle tier of the hierarchy is largely based on future informal bonuses that can be foregone with a higher probability following the deviation. The second key parameter is a proxy for business conditions that affect the organization, represented by the probability of favorable exogenous shocks. Specifically, we interpret favorable business conditions as when the organizational surplus decreases less when agents shirk compared to when business conditions are unfavorable. This means that lowering agents' effort incentives has a less negative effect on the organizational surplus as business conditions become more favorable. The flipside is that the principal's incentives to deviate against agents increase in the favorable state. From these discussions, we can deduce the following results. First, hierarchy is an optimal organizational structure when the level of trust is high and business conditions are favorable. Second, hierarchy cannot be optimal when the level of trust is sufficiently low. Third, when business conditions are sufficiently favorable, centralization cannot achieve the first best regardless of the level of trust, because of the principal's strong deviation incentives. Fourth, when business conditions are unfavorable, centralization can be optimal even if hierarchy is not. In sum, hierarchy can be an optimal organizational structure to complement long-term relationships when trading parties have a high level of trust and business conditions are favorable.

We take our theoretical predictions to examine relational contracts that govern supplier networks in the automotive industry. It is well known that the Japanese automakers such as Toyota and Honda organize their supplier networks based on relational contracts in a hierarchical way. For example, Toyota has a limited number of first-tier suppliers that belong to an association called *Kyohokai*, with whom it maintains long-term relationships. Moreover, Toyota outsources multiple tasks such as the design and production of various car parts to its first-tier suppliers, who then use second-tier supplies as subcontractors, who in turn enter into subcontracting relationships with third-tier suppliers, and so on (Asanuma, 1988; Nishiguchi, 1994; Fujimoto, 1999). In contrast, the US automakers such as GM and Ford traditionally had supplier networks that are less hierarchical, and they directly contracted with many suppliers via arm's length contracts without relying on long-term relationships (Asanuma, 1988; McMillan, 1990; Taylor and Wiggins, 1997). But the US automakers underwent substantial restructuring in the 1980s, which introduced various elements of Japanese-style relational contracting such as long-term relationships and hierarchical subcontracting. We can apply our theory to compare the two contrasting models of supplier networks and also to critically assess the restructuring of the US automotive industry in the 1980s.

A large body of empirical and anecdotal evidence reports a stark difference in the level of trust in the buyer-supplier relationships between the Japanese and US automotive industries (Dyer, 1997; Dyer and Chu, 2003; Liker and Choi, 2004; Helper and Henderson, 2014). For example, Dyer and Ouchi (1993) report survey results that show a high level of confidence suppliers have in their buyers in Japan: suppliers indicated over 90 percent probability of winning the contact again, with Toyota's and Nissan's suppliers having essentially open-ended contracts. In contrast, Dyer (1997) reports that GM's procurement costs were six to eight times higher than Toyota's mainly because suppliers viewed GM as a much less trustworthy organization. This suggests a high level of trust in the Japanese automotive industry, relative to its US counterpart. In addition, one may say the high-growth period of 1960s - 1980s in the Japanese economy represents favorable business conditions facing automakers. As our theory predicts, hierarchy is an optimal structure in this case, which can be one of the factors that explain Toyota's success. While several authors attribute Toyota's success to its long-term and cooperative relationships with its suppliers based on mutual trust (Helper and Sako, 1995; Helper and Henderson, 2014), such relationships can be better complemented by hierarchical, rather than centralized, supplier networks. On the other hand, given the low level of trust, the Japanese-style restructuring of the US automakers in the 1980s, GM in particular, was ill-fated (Liker and Choi, 2004; Helper and Henderson, 2014). As Helper and Henderson (2014) argue, GM's priority should have been building trust-based relationships with suppliers. As our theory predicts, hierarchy cannot be optimal when the level of trust is sufficiently low.

The remainder of the paper is organized as follows. After briefly explaining how our work contributes to the related literature below, we describe the model in Section 2. In Section 3, we characterize necessary and sufficient conditions for the first best under each organizational structure. In Section 4, we compare the two organizational structures and derive testable predictions, which are discussed in the context of the automotive industry in Section 5. Section 6 contains concluding remarks. We provide the sketch of proofs in the Appendix, while relegating more details to the Online Appendix.

Related Literature

Our paper makes novel contributions to three strands of literature. First, we contribute to the literature on organization design in general, and hierarchies in particular. Second, we add to the literature on relational contracting with multiple parties by showing when hierarchy can be an optimal way to manage relational contracts. Third, we make theoretical contributions to the management literature on supplier networks. We provide below a brief review of each strand of literature.

The existing literature studies hierarchical organizations from several different angles. The first group of studies has focused on the features of hierarchies such as information processing, loss of control, allocation of tasks, and so forth, by taking the hierarchical structure as given (e.g., Williamson, 1967; Radner, 1992; Bolton and Dewatripont, 1994; Chen and Suen, 2019). The second group has studied when hierarchy can be an optimal organizational structure among all possible organizational structures. Because centralization cannot be dominated by any other organizational structures in an environment where the revelation principle applies, these studies depart from such an environment by considering, for example, communication costs or collusion between agents, and show when hierarchy can dominate centralization (e.g., Melumad et al., 1995; Mookherjee and Tsumagari, 2004; Mookherjee, 2006; Choe and Park, 2011). The third group of studies adopts an incomplete contracting approach, where the allocation of authority is the central issue (Hart and Moore, 2005; Choe and Ishiguro, 2012; Ishida, 2015, Kräkel, 2017). All of these studies are in a static setting, whereas our model is dynamic and our focus is on conditions that guarantee the efficiency and stability of different organizational structures.

There are a number of studies on relational contracts with multiple agents (Levin, 2002; Kvalöy and Olsen, 2006; Rayo, 2007; Board, 2011; Ishihara, 2017; Troya-Martinez and Wren-Lewis, 2021). When multiple agents can mutually observe their performance signals and payments to each other, they can resort to *multilateral punishment* against the principal if the principal deviates against at least one of them (Levin, 2002). Andrews and Barron (2016) and Barron and Powell (2019) drop this assumption by allowing secret deviations and study dynamic policies that govern centralized relational contracts. We also drop the assumption of multilateral punishment but address the different question of when transactions can be better organized in a hierarchical or a centralized way. This question has broader relevance to dynamic incentives and organization design when verifiable information is limited so that parties must rely on relational contracting. Troya-Martinez and Wren-Lewis (2021) consider a three-tier relational contracting model in which the principal lets the manager contract with the agent. But the principal is not an active player in their model in that she does not make any strategic choice in all subsequent periods after the initial period. Furthermore, they do not compare hierarchy with centralization in terms of efficiency, which is what we do in this paper.

Finally, a large body of literature in management has been devoted to studying the Japanese-style, hierarchical, relational contracting vis-à-vis the US-style, centralized, arm's length contracting in supplier networks, with a particular focus on the automotive industry (e.g., Asanuma, 1988; Clark and Fujimoto, 1991; Cusumano and Takeishi, 1991; Helper and Sako, 1995; Dyer, 2000; Linker and Choi, 2004). Our paper provides theoretical underpinnings to this literature by highlighting trust and the underlying business environment as crucial factors in the design of supplier networks based on long-term relationships.

2 Model

2.1 Production and Payoffs

Time is discrete and extended over infinity, indexed by $t \in \{1, 2, 3, ...\}$. There are three players, a principal (she) and agents 1 and 2 (he), who are all risk neutral and without limited liability. All players have reservation utility normalized to zero and they discount future payoffs using the common discount factor $\delta \in [0, 1)$. In period t, agent i chooses an unobservable effort $a_{i,t} \in \{0, 1\}$ at personal cost $c(a_{i,t})$ where $c(1) \equiv c > c(0) \equiv 0$. Given a pair of efforts $(a_{1,t}, a_{2,t})$, the principal obtains stochastic output $y_t \in \{Y, 0\}$ in period

t where $Y > 0$. More specifically, output Y is realized with probability $p(a_{1,t}, a_{2,t}) \equiv \Pr(y = Y | a_{1,t}, a_{2,t}) \in [0, 1]$. To simplify notation and analysis, we assume $p(0, 1) = p(1, 0)$, $\Delta p \equiv p(1, 1) - p(0, 1) > 0$ and $p(0, 0) = 0$. The realized output in period t is observable to all players but not verifiable, which makes it impossible to write formal contracts based on y_t.

In each period, the joint surplus is the sum of all players' payoffs, denoted by $s(a_1, a_2) \equiv p(a_1, a_2)Y - \sum_i c(a_i)$ where we omitted the time subscript. We use the following assumption throughout the paper.

Assumption 1. *$Y \geq c/\Delta p$ and $Y \geq 2c/p(1, 1)$.*

Assumption 1 implies $s(1, 1) \geq \max\{s(1, 0), s(0, 1), s(0, 0)\}$ and, therefore, $(a_1, a_2) = (1, 1)$ maximizes $s(a_1, a_2)$. Thus we can define the first-best surplus as

$$s^* \equiv s(1, 1) = p(1, 1)Y - 2c, \tag{1}$$

and the discounted present value of the first-best surplus as

$$S^* \equiv \frac{s^*}{1 - \delta}.$$

2.2 Organizational Structures

There are two possible organizational structures in allocating contracting authorities among the three players: centralization and hierarchy. Under centralization, the principal directly contracts with both agents (as illustrated in the left panel of Figure 1). Under hierarchy, the principal contracts with only one of the agents, say agent 1, who is delegated the authority to contract with the other agent. In this case, agent 1 is at the middle tier of the hierarchy and agent 2 is at the bottom tier (as illustrated in the right panel of Figure 1).

We make two assumptions about the payments used under both organizational structures. First, as in Andrews and Barron (2016), we assume that payments are made *secretly*. By secret payments, we mean that only the two parties directly involved in a contractual relationship can observe the full history of payments between them; the remaining party who is outside the contractual relationship is not privy to such information. If the history is publicly observed by all players, then the two organizational structures are equivalent in the sense that both organizational structures have the same necessary and sufficient conditions for the first best, as we show in the Appendix.

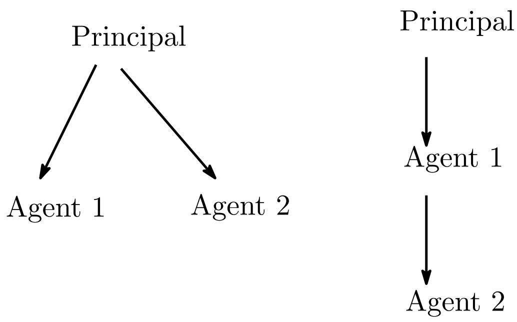

Figure 1

Second, we assume that the party with contracting right to trade with agent i has the "default option" to exercise against agent i. When exercised in period t, the default option nullifies agent i's contribution to high output in that probability $p(a_{i,t}, a_{j,t})$ changes to $p(0, a_{j,t})$. Under centralization, the principal can exercise the default option against either agent, potentially altering $p(a_{i,t}, a_{j,t})$ to $p(0, a_{j,t})$ or $p(a_{i,t}, 0)$. Under hierarchy with agent i in the middle tier, the principal has the default option against agent i, who has the default option against agent j. We assume that the exercise of default option is also observed only by the two parties directly involved in a contractual relationship. The default option has a natural analogue in real-world contracting as we discuss below in assembler-supplier relationships.

As an example that motivates our model, one can think of the principal as a final good assembler (for example, automaker) and agents as suppliers of intermediate goods (for example, automobile parts). Supplier i's effort a_i enhances the quality of intermediate good i which stochastically increases the revenue from the final good $y \in \{Y, 0\}$. Under centralization, the assembler purchases both intermediate goods directly from the suppliers. Under hierarchy, the assembler directly contracts with one of the suppliers, say supplier 1, to deliver both intermediate goods, who in turn contracts with supplier 2 to deliver intermediate good 2. This type of subcontracting relationship is common in many industries that involve a large number of intermediate goods necessary for the final good assembly. For example, about 30,000 parts are needed to manufacture an automobile. Thus one can always find some form of hierarchical subcontracting in automotive industries around world, although hierarchical subcontracting is more pronounced in some countries than in others. As we discuss in depth in Section 5, hierarchical subcontracting is a salient feature of the Japanese automotive industry whereas centralized arm's length contracting is more common in the US counterpart. Understanding the economic factors that explain such differences is one aim of this paper.

We now provide two possible interpretations of the default option in the context of assembler-supplier relationships. For the first interpretation, suppose there are two types of intermediate goods: a standard good for which the supplier's effort is inessential and a customized good that requires the supplier's costly effort $a_{i,t} \in \{0, 1\}$ to improve its quality. In this case, exercising the default option means choosing the standard intermediate good. The second interpretation is in-house production. The party exercising the default option against a supplier produces the intermediate good in-house without relying on the supplier's effort. Given our assumption that the exercise of default option is not observed by those outside the contractual relationship, the first interpretation may be deemed more plausible than the second one.

2.3 Timing

We first consider centralization. The timing within each period t is as follows.

1. The principal and agents simultaneously make up-front transfers, denoted by $w^p_{i,t} \geq 0$ and $w^i_{p,t} \geq 0$, where $w^i_{p,t} \geq 0$ is the principal's transfer to agent i and $w^p_{i,t} \geq 0$ is agent i's transfer to the principal. Agent $j \neq i$ cannot observe $(w^i_{p,t}, w^p_{i,t})$. We denote by $w^i_t \equiv w^i_{p,t} - w^p_{i,t}$ the net up-front transfer agent i receives from the principal.

2. The principal decides whether or not to exercise the default option against either agent. Whether the principal has exercised it against agent i is observed only by agent i.[1] We denote by $d^i_{p,t} \in \{0, 1\}$ the principal's decision to exercise the default option against agent i where $d^i_{p,t} = 0$ when the default option is exercised.

3. Agents choose efforts $a_{i,t} \in \{0, 1\}$ simultaneously.

4. Output $y_t \in \{Y, 0\}$ is realized and observed by all parties.

5. The principal and agents simultaneously make ex-post transfers (called *informal bonuses*), denoted by $b^p_{i,t} \geq 0$ and $b^i_{p,t} \geq 0$, where $b^i_{p,t} \geq 0$ is the principal's payment to agent i and $b^p_{i,t} \geq 0$ is agent i's payment to the principal. Agent $j \neq i$ cannot observe $(b^p_{i,t}, b^i_{p,t})$. We denote by $b^i_t \equiv b^i_{p,t} - b^p_{i,t}$ the net informal bonus agent i receives from the principal.

As explained previously, the full history of a contractual relationship can be observed only by the parties directly involved in the contractual relationship. Under centralization, this means that agent i cannot observe the history of agent j's transactions with the principal. Moreover, we follow Andrews and Barron (2016) to assume that agents cannot communicate with each other to share information about their past transactions with the principal.[2]

Next we consider hierarchy. Because agents are homogeneous, we assume without loss of generality that agent 1 is assigned at the middle tier of the hierarchy.

1. The principal and agents simultaneously make up-front transfers to each other. The transfers between the principal and agent 1 are denoted by $w^1_{p,t}$ and $w^p_{1,t}$ as before,

[1] For simplicity, we suppress the participation decisions by the players. Since the reservation payoff is zero, the principal's participation constraint is satisfied by the default option. Also, agents can ensure the reservation payoff of zero by paying nothing and choosing low effort. Even when we explicitly consider the participation decisions, our results stay qualitatively the same as long as the participation decisions are observed only by the parties directly involved in the transaction in question, as assumed in Andrews and Barron (2016).

[2] We discuss the case of observable transfers in the Online Appendix.

and the transfer from agent i to agent j is denoted by $w_{i,t}^{j}$. Agent 2 cannot observe the transfers between the principal and agent 1, $(w_{p,t}^{1},\, w_{1,t}^{p})$. The principal cannot observe the transfers between agents 1 and 2, $(w_{1,t}^{2},\, w_{2,t}^{1})$. As before, denote the net up-front transfer for agent i by w_t^i: $w_t^1 = w_{p,t}^1 - w_{1,t}^p$ and $w_t^2 = w_{1,t}^2 - w_{2,t}^1$.

2. The principal makes a decision to exercise the default option against agent 1, denoted by $d_{p,t}^1 \in \{0,1\}$, and agent 1 makes a decision to exercise the default option against agent 2, denoted by $d_{1,t}^2 \in \{0,1\}$. The exercise of default option is privately observed only by the parties in the relevant transaction.

3. Agents choose efforts $a_{i,t} \in \{0,1\}$ simultaneously.

4. Output $y_t \in \{Y, 0\}$ is realized and observed by all parties.

5. The principal and agents simultaneously pay informal bonuses to each other: the principal pays $b_{p,t}^1 \geq 0$ to agent 1, agent 1 pays $b_{1,t}^p \geq 0$ to the principal and $b_{1,t}^2 \geq 0$ to agent 2, and agent 2 pays $b_{2,t}^1 \geq 0$ to agent 1. Agent 2 cannot observe $(b_{p,t}^1,\, b_{1,t}^p)$, and the principal cannot observe $(b_{1,t}^2,\, b_{2,t}^1)$. As before, denote the net informal bonus for agent i by b_t^i: $b_t^1 = b_{p,t}^1 - b_{1,t}^p$ and $b_t^2 = b_{1,t}^2 - b_{2,t}^1$.

As in centralization, only the parties directly involved in the contractual relationship can observe the full history of their transactions. Under hierarchy, this means that the principal cannot observe the history of the transactions between agents 1 and 2, and agent 2 cannot observe the history of agent 1's transactions with the principal. In addition, the principal and agent 2 cannot communicate with each other to share their private information.

From the above, we can highlight the main difference between centralization and hierarchy as follows. Under centralization, the principal is the key player in the network of transactions and holds all the relevant information. Under hierarchy, it is the agent at the middle tier, agent 1 by our assumption, who holds all the relevant information. As we discuss below, the relative performance of each organizational structure depends crucially on controlling the deviation incentives by the party who has all the relevant information. Thus centralization can outperform hierarchy in an environment where the principal's deviation incentives are weak. But hierarchy can outperform centralization if the principal can control the deviation incentives by the agent at the middle tier at relatively low cost.

3 Equilibrium Characterization

In this section, we characterize the necessary and sufficient conditions for the first-best surplus $S^* = s^*/(1-\delta)$ to be supported in period 1 under each organizational structure.

3.1 Centralization

We begin with centralization. We need to introduce some notation. Let $h_{0,k}^t$ denote a history observed by player $k \in \{1, 2, p\}$ up to the beginning of period t, where $k = p$ denotes the principal. For example, in the beginning of period t, the principal observes a history

$$h_{0,p}^t = h_{0,p}^{t-1} \cup \{\{w_{p,t-1}^i\}_i, \{w_{i,t-1}^p\}_i, \{b_{p,t-1}^i\}_i, \{b_{i,t-1}^p\}_i, \{d_{p,t-1}^i\}_i, y_{t-1}\},$$

and agent i observes a history

$$h_{0,i}^t = h_{0,i}^{t-1} \cup \{w_{i,t-1}^p, w_{p,t-1}^i, b_{i,t-1}^p, b_{p,t-1}^i, d_{p,t-1}^i, a_{i,t-1}, y_{t-1}\}.$$

Note that $h_{0,k}^1 \equiv \emptyset$ for $k \in \{1, 2, p\}$.

Denote by $\mathcal{A} = \{w, d, a, y, b\}$ the set of events that occur within a given period: $w \in \mathcal{A}$ denotes the event that up-front transfers are made, $d \in \mathcal{A}$ denotes the event that the decision on the default option is made, and so on. Let $H_{e,k}^t$ be the set of all histories leading up to $e \in \mathcal{A}$ in period t observed by player $k \in \{1, 2, p\}$. For example, $h_{y,k}^t \in H_{y,k}^t$ is the history observed by player k leading up to the realization of $y_t \in \{Y, 0\}$. That is, for the principal, we have $h_{y,p}^t = h_{0,p}^t \cup \{\{w_{p,t}^i\}_i, \{w_{i,t}^p\}_i, \{d_{p,t}^i\}_i, y_t\}$. Then the principal's strategy specifies actions at each history $h_{e,p}^t \in H_{e,p}^t$ for each $e \in \mathcal{A} \cup \{0\}$ where 0 indicates the beginning of a period. Likewise, agent i's strategy specifies actions feasible at each information set given his belief about the past histories. Note that agent i cannot observe any part of agent j's ($j \neq i$) history $h_{e,j}^t$ except $y^t \equiv (y_1, ..., y_t)$, a history of output realized up to the end of period t.

We focus on perfect Bayesian equilibrium (equilibrium, hereafter) in pure strategies that attains S^*. That is, both agents choose $a_{i,t} = 1$ in all t and the stipulated upfront transfers and informal bonuses are paid on the equilibrium path. In finding the necessary and sufficient conditions for the equilibrium that support the first best, we first derive necessary conditions by checking a subset of possible deviations that can be deemed 'representative'. We then show that these necessary conditions are also sufficient by explicitly constructing strategies and beliefs on- and off-the equilibrium path. We state our first main result below.

Proposition 1. *The first-best surplus $S^* \equiv s^*/(1-\delta)$ is supported under centralization if and only if*

$$\delta S^* \geq \max\left\{\frac{2c}{\Delta p}, \ \frac{c}{\Delta p} + \left(\frac{\delta}{1-\delta}\right)\pi\right\} \tag{2}$$

where $\pi \equiv p(1, 0)Y$ and $\Delta p \equiv p(1, 1) - p(1, 0) > 0$.

Proof. See the Appendix.

A formal proof of the above proposition is long and tedious, as it involves considering all possible deviations and the possibility of non-stationary strategies. We provide below the general intuition for the case where equilibrium net transfers are stationary, that is, $\hat{b}^i \equiv \hat{b}^i_t$ and $\hat{w}^i \equiv \hat{w}^i_t$ for all $t \geq 1$ and $i = 1, 2$, and are independent of the past output history y^{t-1}. Thus, $\hat{b}^i(\{y_t\} \cup y^{t-1}) = \hat{b}^i(y_t)$ for all y^{t-1}. But the intuition holds more generally as shown in the proof. In what follows, we show that the condition in Proposition 1 means that the discounted future value of the joint surplus given in the LHS of (2), δS^*, must not be smaller than the deviation gains the principal can achieve by reneging on informally promised bonus payments to one or both agents, which are given in the RHS of (2).

As explained previously, we consider some representative deviations by the principal, based on which to derive the necessary conditions for the first best. In the proof of Proposition 1, we show that the identified necessary conditions are also sufficient. Since the basic intuition for Proposition 1 can be offered based on the necessary conditions only, we proceed below by focusing on two possible deviations. First, when high output $y_t = Y$ is realized in period t, the principal can deviate from paying equilibrium informal bonuses to both agents, after which she exercises the default options against both of them from period $t+1$. Second, the principal can make the above deviation against only one agent, say agent i, while maintaining the equilibrium contracts with agent $j \neq i$.

First, consider the case where the principal deviates against both agents in period t. This can save total bonus payments $\sum_{i=1,2} \hat{b}^i(Y)$ in period t. Denote by V the discounted present value of the principal's equilibrium payoff, called the *equilibrium value* hereafter. Then, to ensure that the principal cannot benefit from the above deviation, the dynamic enforcement constraint for the principal (DEP) must be satisfied as follows:

$$\delta V \geq \sum_i \hat{b}^i(Y). \tag{DEP}$$

In the above, the LHS is the discounted equilibrium value for the principal and the RHS is the gain the principal can obtain from the deviation, followed by the continuation payoff which is at least zero.

On the other hand, the equilibrium bonus $\hat{b}^i$ must be designed to motivate agent i to choose high effort $a_{i,t} = 1$ every period. That is, it must satisfy agent i's incentive compatibility constraint (IC_i):

$$\hat{b}^i(Y) \geq \hat{b}^i(0) + \frac{c}{\Delta p} = \frac{c}{\Delta p} \tag{IC$_i$}$$

where we have set $\hat{b}^i(0) = 0$ without loss of generality.

Next, given the assumption that all equilibrium transfers are stationary and $\hat{b}^i(0) = 0$, we can write agent i's discounted present value of equilibrium payoffs as

$$U^i = \hat{w}^i + p(1,1)\hat{b}^i(Y) - c + \delta U^i. \tag{3}$$

Since $U^i \geq 0$, we have $\delta(V + \sum_i U^i) \geq \delta V \geq \sum_i \hat{b}^i(Y) \geq (2c)/\Delta p$ where the second inequality is from DEP and the last inequality is due to IC_i. Since the sum of equilibrium payoffs for all players must be equal to the first-best surplus, i.e., $V + \sum_i U^i = S^*$, we obtain $\delta S^* \geq (2c)/\Delta p$, the first part of (2) in Proposition 1.

Second, the principal can make a secret deviation against only one agent, say agent i, by not paying informal bonus $\hat{b}^i(Y)$ and exercising the default option against him, while maintaining equilibrium contracts with the other agent $j \neq i$ indefinitely. Since agent j cannot observe the deviation by the principal, he still believes that agent i will continue to choose high effort. Consequently, the principal's deviation causes low effort from agent i only, i.e., $a_{i,t} = 0$. Then the principal's payoff per period after the deviation is $p(0,1)Y - p(0,1)\hat{b}^j(Y) - \hat{w}^j = \pi - p(0,1)\hat{b}^j(Y) - \hat{w}^j$. Thus, after the deviation against agent i, the principal can reduce the expected payment to agent j from $p(1,1)\hat{b}^j(Y)+\hat{w}^j$ to $p(0,1)\hat{b}^j(Y)+\hat{w}^j$. Using the expression for the agent's equilibrium expected payoff given in (3), we can rewrite the post-deviation expected payment to agent j as $p(0,1)\hat{b}^j(Y)+\hat{w}^j = (1-\delta)U^j + c - \Delta p\hat{b}^j(Y)$. Since $\hat{b}^j(Y) \geq c/\Delta p$ by IC_j, this expected payment is at most $(1-\delta)U^j$. This implies that the principal's deviation payoff per period is at least $p(0,1)Y - (1-\delta)U^j = \pi - (1-\delta)U^j$. Then, to ensure that the principal cannot gain from the above deviation, we must have

$$\delta V \geq \hat{b}^i(Y) + \frac{\delta}{1-\delta}\{\pi - (1-\delta)U^j\}.$$

By adding $\delta \sum_{k=1,2} U^k$ to both sides of the above inequality and using $U^k \geq 0$ and IC_j, we have $\delta S^* \geq c/\Delta p + \delta\pi/(1-\delta)$, the second part of (2) in Proposition 1.

3.2 Hierarchy

We now turn to hierarchy with agent 1 at the middle tier. As before, let $h^t_{0,k}$ denote a history observed by player $k \in \{1,2,p\}$ up to the beginning of period t and $h^t_{e,k}$, a history observed by player k until stage $e \in \mathcal{A}$ within period t. For example, in the beginning of period t, the principal observes

$$h^t_{0,p} = h^{t-1}_{0,p} \cup \{w^1_{p,t-1}, w^p_{1,t-1}, b^1_{p,t-1}, b^p_{1,t-1}, d^1_{p,t-1}, y_{t-1}\},$$

agent 2 observes

$$h^t_{0,2} = h^{t-1}_{0,2} \cup \{w^2_{1,t-1}, w^1_{2,t-1}, b^2_{1,t-1}, b^1_{2,t-1}, d^2_{1,t-1}, a_{2,t-1}, y_{t-1}\},$$

and agent 1 observes the entire history except the efforts chosen by agent 2: $h^t_{0,1} = h^t_{0,p} \cup h^t_{0,2} \cup a^{t-1}_1 \setminus a^{t-1}_2$ where $a^{t-1}_i = \{a_{i,1}, ..., a_{i,t-1}\}$ is a history of efforts chosen by agent i until period $t-1$. Let $H^t_{e,k}$ be the set of all histories observed by player $k \in \{1,2,p\}$ up to stage $e \in \mathcal{A} \cup \{0\}$ in period t. Then the principal's strategy specifies actions feasible at each information set given her belief about the histories. Agent 1's strategy specifies feasible actions at each history $h^t_{e,1}$ and agent 2's trategy specifies feasible actions at each information set given his belief about histories $h^t_{e,p} \cup h^t_{e,1}$ for each $e \in \mathcal{A} \cup \{0\}$.

Let u be the static equilibrium payoff for an agent in the benchmark where output y is verifiable and the agent is protected by limited liability (LL). Then we have a standard moral hazard contracting problem: $\min_b$ $p(1,1)b$ subject to (IC) $p(1,1)b - c \geq p(1,0)b$ and (LL) $b \geq 0$. This leads to $b = c/\Delta p$, hence

$$u \equiv p(1,1)\frac{c}{\Delta p} - c > 0. \tag{4}$$

We also define Γ^* as follows:

$$\begin{aligned}\Gamma^* &= \min_{\Phi} \ \Big\{c - \delta S^* + (1-p(1,1))\Phi \\ &\quad + \max_{a\in\{0,1\}} \ \{q(a)\max\{\delta S^* + \delta\Gamma^*, 0\} \\ &\quad + (1-q(a))\max\{\delta S^* - \Phi + \delta\Gamma^*, 0\} - c(a)\} \Big\} \end{aligned} \tag{5}$$

subject to $2c/\Delta p \leq \Phi \leq \delta S^* - \delta u$, where $q(a) \equiv p(a, 0)$. We show in the Online Appendix that $\Gamma^* < 0$ and $\Gamma^* \to -\infty$ as $\delta \to 1$. As discussed below, Γ^* is related to agent 1's incentives to deviate against agent 2. The following proposition summarizes the result for hierarchy.

Proposition 2. *The first-best surplus $S^* \equiv s^*/(1-\delta)$ is sustained under hierarchy if and only if*

$$\delta S^* \geq \delta u + \frac{2c}{\Delta p}, \tag{6}$$

and

$$-\frac{c}{\Delta p} \geq \delta\Gamma^*. \tag{7}$$

Proof. See the Appendix.

The crucial difference between centralization and hierarchy is how information is allocated among different players, which in turn affects their deviation incentives and hence the self-enforceability of the first best. Under centralization, the principal has all the relevant information, which allows her to make a secret deviation against one of the agents without triggering the punishment from the other agent. Under hierarchy, it is the agent at the middle tier, agent 1, who has all the relevant information, who can thus make a secret deviation against agent 2 by not paying the informal bonus. Because the principal cannot observe agent 1's deviation against agent 2, she cannot directly punish agent 1. But agent 2 can punish agent 1 by choosing low effort in all future periods, which reduces the probability agent 1 receives the informal bonus from the principal in the future. This implies that the principal can punish agent 1 indirectly by choosing the informal bonus that can be lost with higher probability when agent 1 deviates against agent 2. Thus, controlling agent 1's deviation incentives boils down to designing the contract which ensures that the discounted value of the loss for agent 1 from the deviation is large enough to outweigh the current deviation gain. This implies that the optimal contract gives the largest possible informal bonus to agent 1 as long as the dynamic enforcement constraint for the principal is not violated. We discuss this intuition in more detail below.

As before, we can provide the intuition for the above proposition by focusing on stationary equilibrium.[3] Thus we drop the time subscript t from equilibrium net transfers

[3]We also consider non-stationary equilibria in the formal proof given in the Appendix.

$\{\{\hat{w}_t^i\}_i, \{\hat{b}_t^i\}_i\}_{t=1}^{\infty}$. Then the discounted present value of agent 1's equilibrium payoffs, denoted by $\hat{U}^1$, is given by

$$\hat{U}^1 = \hat{w}^1 + E_y[\hat{b}^1(y)|1,1] - \hat{w}^2 - E_y[\hat{b}^2(y)|1,1] - c + \delta\hat{U}^1. \tag{8}$$

Consider first the contracting problem between agents 1 and 2. It is intuitively clear that agent 1 will choose the contract for agent 2 such that $b^2(Y) = c/\Delta p$ and $b^2(0) = 0$. That is, agent 2 is paid an informal bonus each period that makes his incentive compatibility constraint binding. In addition, agent 1 will choose $\hat{w}^2$ so that agent 2 does not enjoy any rent: $\hat{w}^2 + p(1,1)\hat{b}^2(Y) = c$.

Let us now turn to agent 1. In order to implement the first best, two sets of incentive constraints need to be satisfied. First, the principal should induce agent 1 to choose high effort while committing herself not to deviate against agent 1. Second, agent 1 should be given incentives not to deviate against agent 2. As we discuss below, the first set of incentive constraints yields condition (6), and the second set of incentive constraints leads to condition (7) in Proposition 2.

We start with the first set of constraints. At the end of period t, agent 1's continuation value given output y_t is the sum of his informal bonus and the discounted future value of his continuation payoffs $\hat{U}^1$, hence $\hat{b}^1(y_t) + \delta\hat{U}^1$. We call this the *continuation bonus* in period t. Denote by ϕ the continuation bonus in period t when $y_t = 0$. That is,

$$\phi \equiv \hat{b}^1(0) + \delta\hat{U}^1.$$

Next, we define the *incentive value*, denoted by Φ, as the difference between the continuation bonus when $y_t = Y$ and the continuation bonus when $y_t = 0$:

$$\Phi \equiv \hat{b}^1(Y) + \delta\hat{U}^1 - \phi.$$

As we discuss below, the incentive value Φ plays an important role in determining agent 1's effort incentives in period t.

Condition (6) in Proposition 2 is obtained from the constraints on the incentive value Φ. First, agent 1 chooses high effort if and only if the incentive value Φ covers his own incentive cost $c/\Delta p$ plus the bonus payment to agent 2 to induce high effort, which is equal to $c/\Delta p$, as discussed earlier. This gives us the lower bound for the incentive value: $\Phi \geq 2c/\Delta p$. Second, for the principal's commitment not to deviate against agent 1, the incentive value Φ must not be too large. Because agent 1 has an informational advantage over the principal under hierarchy, he enjoys some information rent as his equilibrium value, that is, $\hat{U}^1 \geq u$. We show in the Appendix (Lemma A5) that the information rent is not smaller than the static equilibrium payoff given in (4). This implies that the principal's equilibrium value is at most $S^* - u$. Thus the discounted continuation value of the principal $\delta(S^* - u)$ must not be less than the incentive value Φ. Otherwise, the principal will renege on paying Φ to agent 1, hence we have an upper bound for the incentive value $\Phi \leq \delta(S^* - u)$. Combining these two inequalities leads us to condition (6) in Proposition 2.

Next, we will show that condition (7) in Proposition 2 must be satisfied to ensure that agent 1 cannot make a profitable deviation against agent 2. Suppose agent 1 deviates against agent 2 by not paying the informal bonus in period t and exercising the default option thereafter. This deviation is followed by agent 2 choosing low effort forever. However, since the principal cannot observe such a deviation, agent 1 can continue to maintain the equilibrium relational contract with the principal. Let $\tilde{U}^1$ be the discounted present value of continuation payoffs that agent 1 can obtain from the above deviation. Agent 1 has two options after the deviation: (i) he continues to receive the equilibrium transfers $\{\hat{w}^1, \hat{b}^1\}$ from the principal, which gives him the continuation value, $\hat{b}^1(y_s) + \delta\tilde{U}^1$, for $s \geq t+1$ or (ii) he can reject these transfers and obtain the payoff of zero indefinitely. By choosing the optimal option and making the optimal effort choice, agent 1 can guarantee at least the deviation value $\tilde{U}^1$ defined as follows:

$$\tilde{U}^1 = \hat{w}^1 + \max_{a\in\{0,1\}} \left\{E_y[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a, 0] - c(a)\right\}.$$

For the above secret deviation against agent 2 not to be profitable, it must be that

$$-\hat{b}^2(y_t) + \delta\hat{U}^1 \geq \delta\tilde{U}^1 \tag{9}$$

for any $y_t \in \{0, Y\}$. Denote the net gain from the deviation by $\Gamma \equiv \tilde{U}^1 - \hat{U}^1$, so that we can express (9) as $-\hat{b}^2(y_t) \geq \delta\Gamma$. Using the definition of $\hat{U}^1$ in (8), we then obtain

$$\Gamma = c - p(1,1)\Phi - \phi + \max_a \left\{q(a)\max\{\Phi + \phi + \delta\Gamma, 0\} + (1 - q(a))\max\{\phi + \delta\Gamma, 0\} - c(a)\right\}$$

where $q(a) \equiv p(a, 0)$.

In implementing the first best, the net gain from the deviation Γ should be minimized. Then, the continuation bonus ϕ corresponding to low output $y_t = 0$ should be set as large as possible under the dynamic enforcement constraint:

$$\delta S^* \geq \Phi + \phi = \hat{b}^1(Y) + \delta\hat{U}^1.$$

This requires that the discounted value of the joint surplus δS^* cannot be lower than the continuation bonus $\hat{b}^1(Y) + \delta\hat{U}^1$ when high output $y_t = Y$ is realized. Otherwise, the principal will renege on paying such continuation bonus to agent 1. Thus it is optimal to set $\delta S^* = \Phi + \phi$ in order to minimize the net deviation gain Γ. Substituting this into Γ, we obtain

$$\begin{aligned}\Gamma \quad = \quad & c - \delta S^* + (1 - p(1,1))\Phi \\ & + \max_a \ \{q(a)\max\{\delta S^* + \delta\Gamma, 0\} \\ & + (1 - q(a))\max\{\delta S^* - \Phi + \delta\Gamma, 0\} - c(a)\} .\end{aligned}$$

We minimize the above net deviation gain Γ by choosing the incentive value Φ optimally subject to (6), which gives us the minimized value Γ^* as defined in (5). This leads to $-\hat{b}^2(Y) = -c/\Delta p \geq \delta\Gamma^*$, condition (7) of Proposition 2.

The optimal incentive value, denoted by Φ^*, is a unique minimizer attaining Γ^*. When δ increases, Φ^* can increase because the constraint $\Phi^* \leq \delta S^* - \delta u$ can be relaxed when S^* increases. Thus, when players become more patient, larger incentive values can be used to make agent 1's deviation less profitable. The larger the incentive value becomes, the more agent 1 has to lose after deviation because the punishment from agent 2 reduces the probability that agent 1 will receive the incentive value in the future. This suggests that hierarchy is less vulnerable to secret deviations than centralization when players are sufficiently patient, as we will see in the next section.

4 Optimal Organizational Structure: Centralization vs. Hierarchy

In this section, we investigate which organizational structure performs better in the sense that it supports the first best in a given environment as represented by various parameters of our model. Specifically, our focus in on the conditions that ensure relevant players do not have incentives to deviate from the equilibrium that supports the first best. We call an organizational structure *optimal* when it attains the discounted present value of the first-best surplus S^* in period 1. As the analysis in the previous section shows, the key relevant player is the principal under centralization and agent 1, the agent at the middle tier, under hierarchy.

Let us start with centralization. Suppose the principal reneges on the informal bonus to one agent, say agent 2, but maintains the equilibrium relational contract with agent 1. Such a secret deviation reduces the principal's expected payment of informal bonus to agent 1, because agent 1 cannot observe the principal's deviation and, therefore, continues to choose the equilibrium level of effort. From the deviation, the principal makes an immediate gain of $b^2(Y) - b^2(0) = c/\Delta p$, the informal bonus to agent 2, and $\pi \equiv p(0,1)Y$ in each period after the secret deviation. Thus the net gain from the deviation is

$$\frac{c}{\Delta p} + \frac{\delta}{1-\delta}(\pi - s^*)$$

by comparing with the future equilibrium surplus $S^* \equiv \delta s^*/(1-\delta)$. If $\pi - s^* = 2c - \Delta p Y > 0$, then the net gain from the secret deviation is positive for any $\delta \in [0, 1)$ so that centralization cannot implement the first best for any value of δ.

Under hierarchy with agent 1 at the middle tier, agent 1 can deviate against agent 2 by not paying the informal bonus $b^2(Y) = c/\Delta p$ and then exercising the default option against him indefinitely. The cost of the deviation is the lower probability of receiving the incentive value Φ^* from the principal in all periods after the deviation. The probability to obtain the incentive value Φ^* is given by $p(a_{1,t}, 0)$ because agent 2 chooses only low effort $a_{2,t} = 0$ indefinitely following the deviation. This gives agent 1 the discounted future loss of $-\delta\Gamma^*$. As we discussed in Section 3.2, Γ^* tends to $-\infty$ as $\delta \to 1$, i.e., players become more patient. Thus, $-\delta\Gamma^* \geq c/\Delta p$ is more likely to hold as $\delta \to 1$. In view of our discussion in the previous paragraph, we can then conclude that, when players are sufficiently patient, hierarchy can attain the first best although centralization cannot.

In order to derive richer implications about the optimal organizational structure, we use the following specification for the probability of high output Y:

$$p(a_1, a_2) \equiv \gamma f(a_1, a_2) + (1-\gamma) g(a_1, a_2) \tag{10}$$

where $\gamma \in (0,1)$, $f(a_1, a_2) \in (0,1)$ and $g(a_1, a_2) \in (0,1)$, respectively. We make the following assumptions.

Assumption 2. *(i) $f(a_1, a_2) > g(a_1, a_2)$ for all $(a_1, a_2) \neq (0,0)$ and $f(0,0) = q(0,0) = 0$. (ii) $\Delta g \equiv g(1,1) - g(0,1) > \Delta f \equiv f(1,1) - f(0,1) > 0$.*

Assumption 3. *$f(1,1)Y > 2c > \Delta f Y \geq c$.*

We can interpret the specification in (10) and the above assumptions as follows. Suppose there are two states of the world relevant to the business environment, called "favorable state" or "unfavorable state", that affect the organization, with the favorable state occurring with probability γ. From (10), the probability of high output Y is $f(a_1, a_2)$ in the favorable state, and $g(a_1, a_2)$ in the unfavorable state. Then, Assumption 2(i) means that high output is realized with a higher probability when the state is favorable, conditional on $(a_1, a_2) \neq (0,0)$. In addition, the ex ante probability of high output, $p(a_1, a_2)$, increases as the business environment becomes more favorable, i.e., γ increases, conditional on $(a_1, a_2) \neq (0,0)$. Assumption 2(ii) relates to the importance of agents' efforts in different business environments. When one agent shirks by choosing low effort while the other agent chooses high effort, the decrease in the probability of high output is given by Δf (resp. Δg) in the favorable state (resp. unfavorable state). Then, Assumption 2(ii) means that the negative effect of unilateral shirking is more pronounced when the business environment is unfavorable. Assumption 3 ensures $\Delta p Y \geq c$ for all $\gamma \in [0,1]$. In addition, we have $p(1,1)Y \geq 2c$ for all $\gamma \in [\underline{\gamma}, 1]$ for some $0 \leq \underline{\gamma} < 1$ where $\underline{\gamma} \in [0,1]$ satisfies $\underline{\gamma} f(1,1) + (1-\underline{\gamma}) g(1,1) = 2c/Y$.[4] Thus, given Assumption 3, Assumption 1 holds for all $\gamma \in [\underline{\gamma}, 1]$.

In addition, we interpret the discount factor δ to reflect the level of trust players have on each other. When players have a higher level of trust, they are more likely to form a common belief that the relationship is long-lasting and stable. We believe this interpretation is quite relevant in applying our analytical results to relational contracts that govern supplier networks, which is our focus in the next section. Indeed, there is an extensive literature in management that highlights the importance of trust in the performance of supplier networks (Helper, 1991; Helper and Sako, 1995; Dyer, 1997; Dyer and Chu, 2003; Liker and Choi, 2004; Helper and Henderson, 2014).

Given the above assumptions, we are ready to compare conditions under which each organizational structure becomes optimal. Recall that we call an organizational structure optimal if it attains the first-best surplus $S^* \equiv s^*/(1-\delta)$ in period $t = 1$. Our focus

[4] In the case of $g(1,1) \geq 2c/Y$, we define $\underline{\gamma} = 0$.

is on the critical value of the discount factor δ that supports the first best under each organizational structure. Let $\delta_C(\gamma) \in (0,1)$ be the critical value of δ above which the first-best surplus is attained under centralization, and let $\delta_H(\gamma) \in (0,1)$ be the corresponding critical value under hierarchy, where we have indicated that these critical values depend on the probability of the favorable shock γ. Define $\gamma^* \in [0,1]$ such that $\Delta pY = 2c$.[5]

Proposition 3. *Suppose Assumptions 2 and 3 hold. Then there exists* $\overline{\Delta g} \in (2c/Y, 1)$ *such that an optimal organizational structure can be characterized as follows.*

(i) *Suppose* $2c/Y \geq \Delta g$. *Then for all* $\gamma \in [\underline{\gamma}, 1]$, *we have* $\delta_H(\gamma) < 1$, *hence hierarchy is optimal for all* $\delta \in [\delta_H(\gamma), 1)$. *But centralization is never optimal for all* $\delta \in (0,1)$ *and all* $\gamma \in [0,1]$.

(ii) *Suppose* $2c/Y < \Delta g \leq \overline{\Delta g}$. *Then for all* $\gamma \in [0,1]$, *we have* $\delta_H(\gamma) < \delta_C(\gamma) < 1$, *hence both organizational structures are optimal for all* $\delta \in [\delta_C(\gamma), 1)$, *but only hierarchy is optimal for all* $\delta \in [\delta_H(\gamma), \delta_C(\gamma))$.

(iii) *Suppose* $\Delta g > \overline{\Delta g}$. *Then there exists* $\tilde{\gamma} \in (0, \gamma^*)$ *such that* $\delta_C(\gamma) < \delta_H(\gamma)$ *for all* $\gamma \in [0, \tilde{\gamma}]$ *but* $\delta_C(\gamma) > \delta_H(\gamma)$ *for all* $\gamma \in [\gamma^*, 1]$. *That is, only centralization is optimal for all* $\delta \in [\delta_C(\gamma), \delta_H(\gamma))$ *and all* $\gamma \in [0, \tilde{\gamma})$, *and only hierarchy is optimal for all* $\delta \in [\delta_H(\gamma), 1)$ *and all* $\gamma \in [\gamma^*, 1]$.

Proof. See the Appendix.

The key parameter of interest in the above proposition is Δg. Recall that Δg measures how much unilateral shirking by an agent decreases the probability of high output in the unfavorable state. Moreover, $\Delta g > \Delta f$ (Assumption 2(ii)) means that the negative effect of unilateral shirking is more severe in the unfavorable state. Thus the principal is less concerned about the unilateral shirking by an agent when Δg is smaller, or γ is larger. In this case, the principal's deviation against an agent becomes more likely: the principal can earn a larger gain from the secret deviation under centralization, that is, $\pi - s^* = 2c - \Delta pY$ is larger, as γ is larger. The above proposition formalizes this intuition: when Δg is small, hierarchy can be optimal but centralization is never optimal; when Δg is large, centralization can be optimal if γ is not large enough, and hierarchy can be optimal, otherwise.

We illustrate Proposition 3 in the three figures with δ on the vertical axis and γ on the horizontal axis. In each figure, we show the range of (γ, δ) that supports the first best under either organizational structure, by plotting the critical values $\delta_C(\gamma)$ and $\delta_H(\gamma)$. We also label different regions by C, H, CH and N, where each of the first three labels indicates the region where one or both organizational structures can attain the first best, and N indicates the region where neither can. We say an organizational structure

[5]If $\Delta gY > 2c$, then $0 < \gamma^* < 1$ given Assumptions 2 and 3. If $\Delta gY \leq 2c$, then we define $\gamma^* = 0$.

dominates the other if the critical discount factor under the former is smaller than the critical discount factor under the latter: in this case, the first best is supported for a wider range of discount factors under the former.

Figure 2 corresponds to the case where Δg is small. In this case, centralization is never optimal, but hierarchy is when the business condition is favorable in that $\gamma \geq \underline{\gamma}$. Region H represents the set of (γ, δ) that supports the first best under hierarchy but neither organizational structure is optimal in region N. Hierarchy dominates centralization in this case. Figure 3 shows the case where Δg is in the intermediate range. In this case, hierarchy once again dominates centralization because the range of (γ, δ) that supports the first best under centralization is a proper subset of that under hierarchy. As shown in Proposition 3, we have $\delta_C(\gamma) > \delta_H(\gamma)$ for all $\gamma \in [0, 1]$, so that region H in Figure 3 represents the range of (δ, γ) where only hierarchy is optimal. Finally, Figure 4 shows the case with large enough Δg. Then centralization can dominate hierarchy for small values of γ: as shown in Figure 4, we have $\delta_C(\gamma) < \delta_H(\gamma)$ for $\gamma \in [0, \tilde{\gamma}]$, so that region C represents the range of (δ, γ) where only centralization is optimal. Thus, when the cost of shirking by an agent is large and the business condition is unfavorable, centralization is a better way to manage relational contracts than hierarchy.

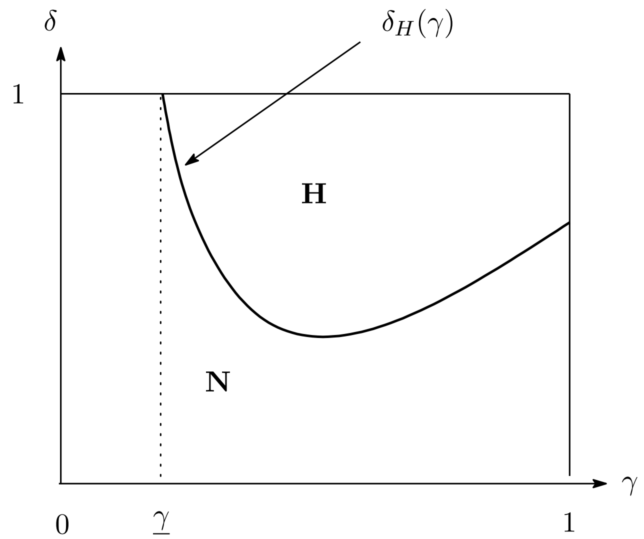

Figure 2

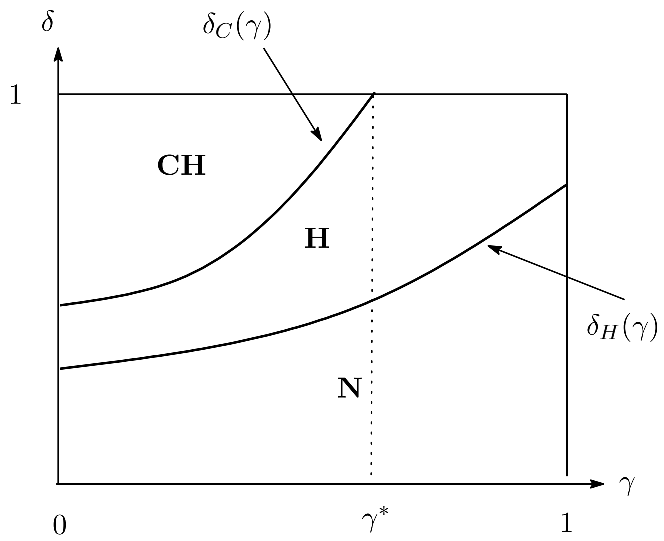

Figure 3

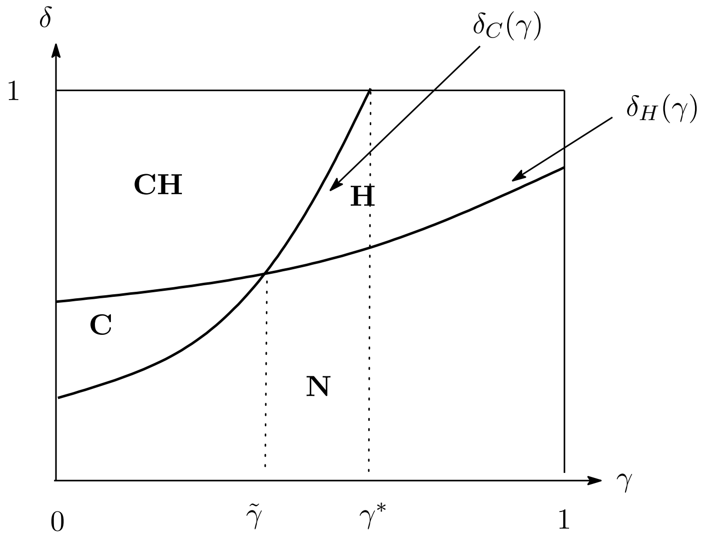

Figure 4

Combining the three cases in Proposition 3, we have the following.

Corollary. *Suppose Assumptions 2 and 3 hold. Then, given $\gamma \in (\gamma^*, 1]$, centralization can never be optimal for any $\delta \in [0, 1)$. Moreover, given $\gamma \in [\underline{\gamma}, 1]$ where $\underline{\gamma} < \gamma^*$, hierarchy* *can be optimal for any* $\delta \in [\delta_H(\gamma), 1)$.

The key implications from our theory are three-fold. First, when business conditions are sufficiently favorable, centralization cannot be optimal regardless of the level of trust. Second, as business conditions improve and the level of trust increases, hierarchy dominates centralization. Third, if the level of trust is low enough, then neither organizational structure can be optimal regardless of business conditions. In the next section, we discuss these implications in the context of supplier networks in the automotive industry.

Remark. When $\delta < \min\{\delta_C(\gamma), \delta_H(\gamma)\}$, neither centralization nor hierarchy achieves the first best. In that case, only $(a_i, a_j) = (1, 0)$ or $(a_i, a_j) = (0, 0)$ is implementable. The latter can be implemented by simply repeating the static equilibrium every period. For the former, suppose the joint surplus $s(1, 0) = p(1, 0)Y - c$ is positive: $s(1, 0) > 0$. Then we can show that the necessary and sufficient condition for $(a_i, a_j) = (1, 0)$ to be implemented becomes $\delta s(1, 0)/(1 - \delta) \geq c/\Delta p$ under both centralization and hierarchy (see the Appendix). The reason is that, when $a_j = 0$, the model is reduced to simple bilateral relational contracting between a principal and agent i. Then we can apply the standard technique (Levin, 2003) to show that centralization and hierarchy are equivalent in terms of the critical discount factor above which $s(1, 0)$ is attained.

5 Implications: Supplier Networks in the Automotive Industry

Our analysis in Section 4 has identified the factors that are central to the stability of hierarchical vs. centralized relational contracting. Two parameters are key in this comparison. First, γ reflects business conditions that affect the firm. When γ is large, the firm is facing fortuitous, favorable business conditions, and unilateral shirking by an agent has a relatively small negative impact. Thus larger values of γ can increase the principal's temptation to deviate from the relational contracting. Second, the discount factor δ relates to the level of trust agents have that the relationships will be long lasting and stable. The larger the value of δ is, the less deviation incentives agents have when delegated the contracting authority. Put together, our theory leads to the following implications on hierarchical relational contracting. First, when the firm is facing favorable business conditions and agents have a high level of trust, hierarchy outperforms centralization. Second, if the level of trust is low enough, then neither organizational structure can be optimal. In this section, we examine these implications in the context of the automotive industry in Japan with a focus on Toyota, and in the United States with a focus on General Motors and Chrysler.

5.1 Toyota

The automotive industry in Japan provides a prime example of vertical *keiretsu*, or a vertical network of firms in an industry, often characterized by long-lasting relationships and repeated transactions based on trust and a continuous flow of information (Goto and Suzumura, 1997). Many studies find that the stability of the network and strong network identity contributed to information sharing and the rapid diffusion of productivity-enhancing knowledge, which proved to be a source of competitive advantage (Nishiguchi, 1994; Branstetter, 2000; Dyer and Nobeoka, 2000). As our theory relates to relational contracting in a dynamic context, vertical *keiretsu* serves as a natural example in which to examine our theoretical predictions. We discuss below Toyota's supplier network in detail, with a special focus on the period of high growth during the 1960s - 1980s.[6]

Toyota's supplier network is divided into hierarchical tiers in an open pyramid structure: major suppliers constitute the first tier, followed by smaller suppliers in successive tiers down the hierarchy. Toyota's first-tier suppliers belong to a group called *Kyohokai* established in 1943. Toyota has direct transactions with only first-tier suppliers, who then transact with second-tier suppliers, which is followed by further subcontracting relationships.[7] Toyota's relationships with its first-tier suppliers entail more than business transactions. Following a *keiretsu* diagnosis in 1952-1953, Toyota became more actively involved in its suppliers' operations including product development, assistance for total quality control, improvement in their management capabilities, etc. First-tier suppliers in turn exercised some control over second-tier suppliers by providing technical assistance and evaluation of their operations. Despite growing demands for its cars and expansion of its production scale through the 1960s, Toyota continued to keep the number of its first-tier suppliers stable. This way, Toyota maintained the multi-tier hierarchical structure of its supplier network, which became firmly established by the 1970s (Wada, 1991). The basic structure remained much the same through the 1990s (Fujimoto, 1999, p. 316).[8]

In a study comparing the supplier networks between the US and Japanese automakers, Cusumano and Takeishi (1991) report that, compared to the US, Japanese automakers rely on a smaller number of suppliers with closer relationships through information exchange and cooperation, and the relationships tend to be longer term and more stable, although without formal guarantees of continued relationships beyond typical contractual periods of 2 to 4 years. Dyer and Ouchi (1993) also report that the Japanese suppliers in their sample indicated over 90 percent probability of winning the contact again, and

[6]For more details on Toyota, its supply chain management and lean production, see, for example, Womack, Jones and Roos (1990), Nishiguchi (1994) or Fujimoto (1999).

[7]A survey by the Small and Medium Enterprises Agency in 1997 reports that Toyota's supplier network comprised 168 first-tier suppliers, 4,000 second-tier suppliers, and 31,600 third-tier suppliers (Wada, 1991). Wada (1991) further reports the results from a 1978 survey that each of the first-tier suppliers used an average of 21.7 second-tier suppliers as its subcontractors, each of the second-tier suppliers used an average of 15.5 third-tier suppliers, and each of the third-tier suppliers were obtaining goods from 3 suppliers on average. As of 2021, there are 224 member companies in *Kyohokai*.

[8]Despite some changes that started around the late 1990s such as increased use of market competition in a bid to reduce costs (Aoki and Lennerfors, 2013), Toyota's supplier network did not change significantly from the multi-tier hierarchical structure.

that Toyota's and Nissan's suppliers had essentially open-ended contracts. The long-term relationships without formal guarantees speak to the relevance of relational contracts. In addition, Toyota tried to build trust of its suppliers in the continuous and stable relationships. Its 1939 "Purchasing Regulations" considered its parts suppliers branch factories and, once a deal was entered into, a permanent deal would be the rule, and that they would work together toward strengthening management abilities and bringing about mutual prosperity (Wada, 1991). In sum, Toyota's suppliers had a high degree of confidence that the relationships will be long term, i.e., a large value of δ in our model.

Finally, the period of 1960s - 1980s is the era of rapid growth in domestic sales and production. For the 20-year period starting from 1960, Japan's real GDP grew annually at about 7% despite the two oil price shocks in the 1970s, and Toyota's sales of passenger cars in the domestic market grew more than 20% per year, with more than one million cars sold for the first time in 1973.[9] Thus we may say this period represents fortuitous, favorable business conditions, indicating a large value of γ.

Given large γ and δ, our theory prescribes hierarchy as an optimal way to organize a network of suppliers when transactions are based on relational contracting. As we have explained above, Toyota's relationship with its suppliers can be best described as hierarchical relational contracting. Thus, our theory lends support for Toyota's supply chain management during the high-growth period of the 1960s - 1980s.

5.2 The US Automotive Industry

In contrast to the practice in Japan, American-style procurement typically relied on competitive bidding with supply contracts awarded to the lowest bidder on an annual basis (Taylor and Wiggins, 1997). Since the 1970s, however, American automakers faced increasing competition from foreign imports, especially from Japan, and saw their competitiveness declining. In response to this, the US automotive industry underwent substantial restructuring in the 1980s. This included the elements of Japanese-style relational contracting such as reducing the number of suppliers, awarding the survivors long-term contracts, and encouraging top-tier suppliers to manage lower-tier suppliers (Liker and Choi, 2004). In addition, GM established in 1984 an automobile company NUMMI (New United Motor Manufacturing, Inc.) as a joint venture with Toyota, as an opportunity to learn about the Toyota Production System.[10] Chrysler established Diamond-Star Motors in 1985 in a joint venture with Mitsubishi, while actively benchmarking Honda for product development and manufacturing (Dyer, 1996). Nevertheless, the US automakers directly transacted with a large number of suppliers. Asanuma (1988) reports that, in

[9]The GDP growth rate is based on the data from World Development Indicators, World Bank, and Toyota's sales growth is calculated using the data available at https://www.toyota-global.com/company/history_of_toyota/75years/data/index.html.

[10]Asanuma (1988, pp 12-13) provides an example explaining how GM reduced the number of suppliers in the 1980s by creating the Japanese-style subcontracting. GM used to buy various parts necessary for the assembly of car seats, and the assembly was done in-house. This involved dealing with eight to ten suppliers. After the change, one of the former suppliers has been upgraded to the seat supplier from whom GM bought seats directly, and the rest have become suppliers to this supplier.

1986, General Motors dealt with approximately 5,500 parts and component suppliers for its North American production, Ford had 2,500 suppliers, and Chrysler, roughly 2,000 suppliers. The corresponding number was 224 for Toyota in 1986, and 163 for Nissan in 1983. To summarize, the 1980s brought elements of Japanese-style relational hierarchy to the US automakers' supplier networks, albeit in a limited way and to differing degrees in different companies (McMillan, 1990).

Despite the restructuring in the 1980s, a large body of empirical and anecdotal evidence suggests that the attempt by American automakers to create the Japanese-style supply chain was not successful in fundamentally altering the buyer-supplier relationships. Surveys conducted in the late 1980s and early 1990s show that suppliers felt lack of commitment from buyers and there continued to exist lack of trust (Helper, 1991; Helper and Sako, 1995). Other comparative studies also show that the level of trust suppliers have in their automaker customers in the US was lower than that in the Japanese automotive industry (Dyer, 1997; Dyer and Chu, 2003; Liker and Choi, 2004; Helper and Henderson, 2014). For example, Dyer (1997) reports that GM's procurement costs were six to eight times higher than Toyota's primarily because suppliers viewed GM as a much less trustworthy organization. Other surveys also show that, among the Big Three automakers in the US, GM was rated as the least trustworthy customer (Liker and Choi, 2004). All of these imply a relatively low level of trust, hence a small value of δ, in the US automotive industry in general, and GM in particular.

Our theory shows hierarchy cannot be an optimal structure when δ is small, regardless of business conditions as reflected in γ. Thus, GM's attempt to introduce the Japanese-style relational hierarchy needed to be supported by nurturing the culture of trust in its supplier network. But there is plenty of evidence that suggests GM failed to establish effective relational contracts with its suppliers, resulting in inefficiency and high defect rates. For example, supplier contributions accounted for one-third of the difference in the Japanese automakers' advantage over the US counterparts in total engineering hours required in developing a new car (Clark and Fujimoto, 1991). Based on a 1990 survey, Cusumano and Takeishi (1991) report that the mean defect rate in supplied parts was 1.81 for the Big Three US automakers, but 0.01 for the five largest Japanese automakers. Interestingly, they report that the mean defect rate for the Japanese transplants in the US, i.e., the US plants managed by the Japanese automakers, was 0.05, also significantly lower than that for the US automakers. In sum, given the low value of δ, GM's restructuring of its supplier network in the Japanese-style relational hierarchy in the 1980s was ill-fated, the point elaborated in detail in Liker and Choi (2004) and Helper and Henderson (2014).

Chrysler appears to have been more successful than GM in restructuring its supplier network into relational hierarchy. Based on the interviews with senior executives at Chrysler and its suppliers during 1993-1996, Dyer (1996) reports how Chrysler improved its relationships with suppliers. In addition to reducing its production supplier base to a little over 1,000 by the early 1990s, Chrysler adopted various elements of relational contracting such as long-term contracts, supplier involvement in product design and development, two-way information flow, profit sharing, etc. But the most important method for building trust was the Supplier Cost Reduction Effort (SCORE), a formal program that committed Chrysler to encouraging, reviewing, and acting on suppliers' ideas quickly and fairly, and to sharing the benefits of those ideas with the suppliers (Dyer, 1996). SCORE turned out to be instrumental in improving communication and coordination, and credibly committing to long-term relationships to suppliers. One may say SCORE helped improve δ for Chrysler, which may partly explain why Chrysler was twice as profitable as GM and Ford in the 1990s (Dyer, 2000). As our theory shows, given large enough δ, hierarchy can always attain the first best but centralization can only if γ is not too large.

6 Conclusion

This paper has studied the problem of optimal organization design in a dynamic setting with one principal and two agents where, due to the lack of verifiable information, players must rely on relational contracts. In centralization, the principal contracts with both agents. In hierarchy, the principal contracts with one agent, who is delegated authority to contract with the other agent. We recapitulate our main findings as follows. First, hierarchy is an optimal organizational structure when the level of trust is high and business conditions are favorable. Second, neither organizational structure is optimal when the level of trust is sufficiently low. Third, when business conditions are sufficiently favorable, centralization cannot be optimal regardless of the level of trust. Fourth, when business conditions are unfavorable, centralization can be optimal even if hierarchy is not. Put together, we have provided a new theory of hierarchy showing that it can be an optimal organizational structure to complement long-term relationships among trading parties who have a high level of trust.

We have applied our theory to evaluate the two contrasting models of supplier networks in the automotive industry in Japan and the US. Central to this discussion is the level of trust suppliers have in their automaker-buyers. A high level of trust demonstrated by Japanese suppliers lends support to hierarchical supplier networks prevalent in Japan. In contrast, the difficulty the US automakers - GM in particular - had in building trust-based relationships with their suppliers implies that their Japanese-style restructuring in the 1980s was going to be less than successful. This may be taken as one of the explanations for the rise of Japanese automakers, Toyota in particular, and the struggle of US automakers, GM in particular, through the latter part of the 20th century.

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7 Appendix: Proofs

In this appendix, we prove the propositions stated in the main text. To save space, we relegate lengthy parts of the proof to the supplementary material in the Online Appendix.

7.1 Proof of Proposition 1

Necessity: Suppose that there exists an equilibrium $\{\{\hat{b}_t^i\}_i, \{\hat{w}_t^i\}_i, \{\hat{d}_{p,t}^i\}_i, \{\hat{a}_{i,t}\}_i\}_{t=1}^{\infty}$ which attains the discounted present value of the first-best surplus $S^* \equiv s^*/(1-\delta)$ in the beginning of $t=1$.

Recall that $h_{0,k}^t$ denotes the history observed by player $k \in \{1,2,p\}$ up to the beginning of period t. Denote the history on the equilibrium path by $\hat{h}_{0,k}^t$, and let $\hat{h}_0^t \equiv \cup_{k=1,2,p}\hat{h}_{0,k}^t$. In the beginning of period t, all players can correctly infer the on-the-path history $\hat{h}_0^t$ because they commonly observe the output history $y^{t-1} = (y_1, ..., y_{t-1})$ and we focus on the pure-strategy equilibrium that attains the first-best surplus S^*. Given $\hat{h}_0^t$, let $V_t(\hat{h}_0^t)$ be the discounted present value of the principal's equilibrium payoff (hereafter, *equilibrium value*) in the beginning of period t. Denote the corresponding equilibrium value for agent i by $U_t^i(\hat{h}_0^t)$ and the discounted present value of the joint surplus by $S_t(\hat{h}_0^t)$. Since $\hat{h}_0^t$ can be correctly inferred from y^{t-1}, we can rewrite them as $S_t(y^{t-1})$, $V_t(y^{t-1})$ and $U_t^i(y^{t-1})$, and $S_t(y^{t-1}) = V_t(y^{t-1}) + \sum_i U_t^i(y^{t-1})$. We also use the notation $U_t^i(y_{t-1}, y^{t-2})$ interchangeably with $U_t^i(y^{t-1})$, and $V_t(y_{t-1}, y^{t-2})$ interchangeably with $V_t(y^{t-1})$. Similarly, let $\hat{h}_{e,k}^t$ denote an on-the-path history observed by player $k \in \{1,2,p\}$ leading up to stage $e \in \mathcal{A}$ within period t, and let $\hat{h}_e^t \equiv \cup_k \hat{h}_{e,k}^t$. Once again, since players can correctly infer $\hat{h}_{e,k}^t$ from the output history, we can write equilibrium transfers as $\hat{w}_t^i(y^{t-1})$ and $\hat{b}_t^i(y^t)$ instead of $\hat{w}_t^i(\hat{h}_0^t)$ and $\hat{b}_t^i(\hat{h}_y^t)$, and so on. We also use the notation $\hat{b}_t^i(y_t, y^{t-1})$ interchangeably with $\hat{b}_t^i(y^t)$.

Denote by $E_{yt}[\,\cdot\,|a_{1,t}, a_{2,t}]$ an expectation over $y_t \in \{0, Y\}$ conditional on the effort profile $(a_{1,t}, a_{2,t})$. Then, $V_t(y^{t-1})$ and $U_t^i(y^{t-1})$ are given by

$$V_t(y^{t-1}) = E_{y_t}\big[y_t - \sum_{i=1,2} \hat{b}_t^i(y_t, y^{t-1})\big|\hat{a}_{1,t}, \hat{a}_{2,t}\big] - \sum_{i=1,2} \hat{w}_t^i(y^{t-1}) + \delta E_{y_t}[V_{t+1}(y_t, y^{t-1})|\hat{a}_{1,t}, \hat{a}_{2,t}],$$

$$U_t^i(y^{t-1}) = E_{y_t}[\hat{b}_t^i(y_t, y^{t-1})|\hat{a}_{1,t}, \hat{a}_{2,t}] + \hat{w}_t^i(y^{t-1}) - c(\hat{a}_{i,t}) + \delta E_{y_t}[U_{t+1}^i(y_t, y^{t-1})|\hat{a}_{1,t}, \hat{a}_{2,t}].$$

Lemma A1. *$S_t(y^{t-1}) = S^*$ and $\hat{a}_{i,t} = 1$ for all $t \geq 1$, all y^{t-1}, and all $i = 1, 2$.*

Proof. Since we have $S_1 = S^*$, it follows that $S^* = s(\hat{a}_{1,1}, \hat{a}_{2,1}) + \delta E_{y_1}[S_2(y_1)|a_{1,1}, a_{2,1}]$. Next, we show $S_2(y_1) = S^*$ for all y_1. Note first that $S_2(y_1) = \sum_{t=2}^{\infty} \delta^{t-2} s(\hat{a}_{1,t}, \hat{a}_{2,t}) \leq \sum_{t=2}^{\infty} \delta^{t-2} s^* = s^*/(1-\delta) = S^*$. Thus $S_2(y_1) \leq S^*$ for all y_1. Suppose now $S_2(y_1) < S^*$ for some y_1. Then we have $(1-\delta)S^* < s(\hat{a}_{1,1}, \hat{a}_{2,1}) \leq s^* = (1-\delta)S^*$, a contradiction. This proves $S_2(y_1) = S^*$ for all y_1. Following the same step but starting with $S_2(y_1) = S^*$, we can show $S_3(y^2) = S^*$. Repeating this, we can show $S_t(y^{t-1}) = S^*$ for all $t \geq 4$. Since $(1-\delta)S^* = s^* = \max_{a_1,a_2} s(a_1, a_2) = s(1, 1)$, it must be that $(a_{1,t}, a_{2,t}) = (1, 1)$ holds for all $t \geq 1$. Q.E.D.

Lemma A2. $\delta S^* \geq 2c/\Delta p$.

Proof. The proof is based on the following deviations by the principal and agents in $t = 1$, combined with agents' incentive compatibility constraints.

First, suppose the principal deviates by reneging on paying $\hat{b}^i_{p,1} \geq 0$ against both agents, refusing to receive $\hat{b}^p_{i,1} \geq 0$ from them in period 1, and exercising the default option against both of them from period 2 indefinitely. Then the principal can save $\sum_i \hat{b}^i_1(y_1)$ in $t = 1$ and obtains nothing thereafter. This leads to the following dynamic enforcement constraint for the principal (DEP) in period 1:

$$\delta V_2(y_1) \geq \sum_i \hat{b}^i_1(y_1) \text{ for all } y_1 \in \{Y, 0\}. \tag{A1}$$

Second, suppose agent i deviates by reneging on paying $\hat{b}^p_{i,1}(y_1) \geq 0$ to the principal, refusing to receive $\hat{b}^i_{p,1}(y_1) \geq 0$ from her in period 1, and choosing low effort and no payments from $t = 2$ forever. This gives agent i continuation payoff of zero. Thus the dynamic enforcement constraint for agent i (DEA) in period 1 must be satisfied as follows:

$$\hat{b}_{i,1}(y_1) + \delta U^i_2(y_1) \geq 0 \text{ for all } y_1 \in \{Y, 0\}. \tag{A2}$$

Third, in order to induce $a_{i,1} = 1$ in $t = 1$, we need the following incentive compatibility constraint for agent i (IC_i):

$$\hat{b}^i_1(Y) - \hat{b}^i_1(0) + \delta U^i_2(Y) - \delta U^i_2(0) \geq c/\Delta p. \tag{A3}$$

Combining (A1) for $y_1 = Y$ together with (A3), we obtain

$$\begin{aligned} \delta\Big\{V_2(Y) + \sum_i U^i_2(Y)\Big\} &\geq \sum_i \{\hat{b}^i_1(Y) + \delta U^i_2(Y)\} \\ &\geq \sum_i \{c/\Delta p + \hat{b}^i_1(0) + \delta U^i_2(0)\} \\ &\geq 2c/\Delta p \end{aligned}$$

where the last inequality follows from (A2). Since $S_2(Y) = V_2(Y) + \sum_i U^i_2(Y)$ must be equal to S^* from Lemma A1, we obtain the desired result. Q.E.D.

Lemma A3. $\delta S^* \geq c/\Delta p + \delta\pi/(1-\delta)$ *where* $\pi \equiv p(1,0)Y$.

Proof. The proof is based on considering the principal's secret deviation against only one of the agents in $t = 1$, combined with agents' incentive compatibility constraints.

Suppose the principal deviates against agent i only in the way described in the proof of Lemma A2, but keeps the equilibrium relational contract with other agent $j \neq i$, hence making net transfers $\hat{b}_t^j(y^t)$ and $\hat{w}_t^j(y^{t-1})$ to agent j. This deviation gives the principal the following continuation value from $t = 2$:

$$\tilde{V}_2(y^2) = p(1,0)Y - E_{y_2}[\hat{b}_2^j(y^2)|1,0] - \hat{w}_2^j(y^1) + \delta E_{y_2}[\tilde{V}_3(y^3)|1,0] \tag{A4}$$

where $a_{i,t} = 0$ for all $t \geq 2$ because the principal exercises the default option against agent i from $t = 2$ onward.

Hereafter, we drop the past output history from the argument in $\hat{b}_t^j$, V_t and U_t^j when there is no confusion, and use the shorthand notation $\hat{b}_t^j(y_t)$, $V_t(y_{t-1})$ and $U_t^j(y_{t-1})$ to denote $\hat{b}_t^j(y_t, y^{t-1})$, $V_t(y_{t-1}, y^{t-2})$ and $U_t^j(y_{t-1}, y^{t-2})$. For agent $j \neq i$, the following IC must be satisfied in $t = 2$:

$$\hat{b}_2^j(Y) + \delta U_3^j(Y) - \{\hat{b}_2^j(0) + \delta U_3^j(0)\} \geq c/\Delta p. \tag{A5}$$

Then, using the definition of $U_t^j(y_{t-1})$ and (A5), the RHS of (A4) can be re-written as

$$\begin{aligned}
& p(1,0)Y - E_{y_2}[\hat{b}_2^j(y_2)|1,0] - U_2^j(y_1) + E_{y_2}[\hat{b}_2^j(y_2)|1,1] - c + \delta E_{y_2}[U_3^j(y_2)|1,1] + \delta E_{y_2}[\tilde{V}_3(y_2)|1,0] \\
= \ & p(1,0)Y - E_{y_2}[\hat{b}_2^j(y_2)|1,0] - U_2^j(y_1) + E_{y_2}[\hat{b}_2^j(y_2)|1,1] - c \\
& + \delta E_{y_2}[U_3^j(y_2)|1,1] + \delta E_{y_2}[\tilde{V}_3(y_2)|1,0] + \delta E_{y_2}[U_3^j(y_2)|1,0] - \delta E_{y_2}[U_3^j(y_2)|1,0] \\
= \ & p(1,0)Y + (p(1,1) - p(1,0))\{\hat{b}_2^j(Y) + \delta U_3^j(Y) - (b_2^j(0) + \delta U_3^j(0))\} - c \\
& + \delta E_{y_2}[\tilde{V}_3^j(y_2) + U_3^j(y_2)|1,0] - U_2^j(y_1) \\
\geq \ & p(1,0)Y + (p(1,1) - p(1,0))(c/\Delta p) - c + \delta E_{y_2}[\tilde{V}_3^j(y_2) + U_3^j(y_2)|1,0] - U_2^j(y_1).
\end{aligned}$$

Thus (A4) implies

$$\tilde{V}_2^j(y_1) + U_2^j(y_1) \geq p(1,0)Y + \delta E_{y_2}[\tilde{V}_3^j(y_2) + U_3^j(y_2)|1,0]. \tag{A6}$$

Substituting $\tilde{V}_s(y_{s-1}) + U_s^j(y_{s-1})$ in the RHS of (A6) over $s \geq 3$ repeatedly, we have

$$\tilde{V}_2^j(y_1) + U_2^j(y_1) \geq \frac{1-\delta^T}{1-\delta} p(1,0)Y + \delta^T E_{y^{T-1}}[\tilde{V}_T^j(y^{T-1}) + U_T^j(y^{T-1})|1,0] \tag{A7}$$

for any $T \geq 1$. Then we must have[11]

$$\tilde{V}_2^j(y^1) + U_2^j(y^1) \geq \frac{p(1,0)Y}{1-\delta}. \tag{A8}$$

Note that we have the following dynamic enforcement constraint for the principal (DEP) in period 1 when $y_1 = Y$ is realized:

$$\delta V_2(Y) \geq \hat{b}_1^i(Y) + \delta \tilde{V}_2^j(Y).$$

Combining the above leads us to

$$\begin{aligned}
\delta\{V_2(Y) + U_2^1(Y) + U_2^2(Y)\} \ \geq \ & \hat{b}_1^i(Y) + \delta U_2^i(Y) + \delta\{\tilde{V}_2^j(Y) + U_2^j(Y)\} \\
\geq \ & c/\Delta p + \hat{b}_1^i(0) + \delta U_2^i(0) + \delta\{\tilde{V}_2^j(Y) + U_2^j(Y)\} \\
\geq \ & \frac{c}{\Delta p} + \left(\frac{\delta}{1-\delta}\right) p(1,0)Y
\end{aligned}$$

[11]If inequality (A8) is reversed, then there exists a large enough T such that $\tilde{V}_2^j(y_1) + U_2^j(y_1) < \frac{1-\delta^T}{1-\delta} p(1,0)Y + \delta^T E[\tilde{V}_T^j(y^{T-1}) + U_T^j(y^{T-1})|1,0]$ because $\tilde{V}_T^j(y^{T-1}) + U_T^j(y^{T-1})$ is bounded. However, this is a contradiction to inequality (A7).

where the second inequality follows from agent i's IC in $t = 1$, and the last inequality is from (A2) and (A8). Since $S_2(Y) = V_2(Y) + \sum_k U_2^k(Y) = S^*$ by Lemma A1, we have the desired result. Q.E.D.

This completes the necessity part of the proof of Proposition 1.

Sufficiency: We show in the Online Appendix that the necessary conditions in Proposition 1 are also sufficient. The basic idea is that players follow the stationary contracts as follows: the principal pays informal bonus $b^i(Y) = c/\Delta p$ and $b^i(0) = 0$ to each agent as long as all players have followed equilibrium contracts in the past; and the up-front transfers are specified as $w^i = c - p(1,1)(c/\Delta p) < 0$ so that agent i pays $-w^i$ to the principal in the beginning of each period insofar as no one deviated in the past. In case of deviation from the equilibrium contract, the principal can exercise the default option against one or both agents. If the conditions in Proposition 1 are satisfied, then neither option is profitable.

7.2 Proof of Proposition 2

Necessity: Suppose there exists an equilibrium $\{\{b_t^i\}_i, \{w_t^i\}_i, d_{p,t}^1, d_{1,t}^2, \{a_{i,t}\}_i\}_{t=1}^\infty$ that attains the discounted present value of the first-best surplus $S^* = s^*/(1-\delta)$ in the beginning of $t = 1$. Note first that Lemma A1 continues to hold. In what follows, we use the notation (y_t, y^{t-1}) interchangeably for y^t whenever necessary. Thus we will write $b_t^i(y^t) = b_t^i(y_t, y^{t-1})$ and $w_t^i(y^{t-1})$ to denote equilibrium informal bonus and up-front transfer paid to agent i in period t, given $y^t = (y_t, y^{t-1})$. Let also denote by $E_{y_t}[\,\cdot\,|a_{1,t}, a_{2,t}]$ the expectation over output y_t in period t conditional on an effort profile $(a_{1,t}, a_{2,t})$ chosen in period t again.

We then denote by $U_t^i(y^{t-1})$ the discounted present value of equilibrium payoffs of agent i (hereafter, equilibrium value) in period t, given an output history y^{t-1} up to the beginning of period t:

$$U_t^i(y^{t-1}) = w_t^i(y^{t-1}) + E_{y_t}[b_t^i(y_t, y^{t-1}) + \delta U_{t+1}^i(y_t, y^{t-1})|\hat{a}_{1,t}, \hat{a}_{2,t}] - c$$

for $i = 1, 2$, given y^{t-1}. We also denote by $V_t(y^{t-1})$ the discounted present value of equilibrium payoffs of the principal (hereafter, equilibrium value) in period t given an output history y^{t-1} up to the beginning of period t:

$$V_t(y^{t-1}) = E_{y_t}\left[y_t - \sum_{i=1,2} b_t^i(y_t, y^{t-1}) + \delta V_{t+1}(y_t, y^{t-1}) \middle| \hat{a}_{1,t}, \hat{a}_{2,t}\right] - \sum_{i=1,2} w_t^i(y^{t-1})$$

given y^{t-1}. Let $S_t(y^{t-1}) \equiv V_t(y^{t-1}) + \sum_{i=1,2} U_t^i(y^{t-1})$ be the joint values of all parties in the beginning of period t.

We begin by showing that we can simplify the contracting problem for agent 2.

Lemma A4. *Suppose there exists an equilibrium that attains S^* under hierarchy. Then* *there exists another equilibrium that attains S^* in which the payments to agent 2 on the equilibrium path are given as follows: for all t and y^{t-1}, $b_t^2(y_t, y^{t-1}) = c/\Delta p$ for $y_t = Y$ and $b_t^2(y_t, y^{t-1}) = 0$ for $y_t = 0$, and $w_t^2(y^{t-1}) = -u$ where $u \equiv p(1,1)c/\Delta p - c$. Thus agent 2 does not enjoy any rent, hence $U_t^2 = 0$ for all $t \geq 1$.*

Proof. See the Online Appendix.

By Lemma A4, we can set $U_t^2 = 0$, $b_t^2(Y, y^{t-1}) = c/\Delta p$, $b_t^2(0, y^{t-1}) = 0$, and $w_t^2(y^{t-1}) = -u$. We use these results below.

Lemma A5. $b_t^1(y^t) + \delta \sum_i U_{t+1}^i(y^t) \geq \delta u$.

Proof. The proof is based on considering the following deviation by agent 1 after the realization of y_t. Suppose agent 1 rejects $b_t^1(y^t)$ from the principal at the end of period t and, in period $t+1$, exercises the default option against agent 2 after the up-front transfer of $w_{t+1}^2(y^t)$ has been made. This deviation gives agent 1 the continuation value of $-w_{t+1}^2(y^t)$ from period $t+1$. Evaluating this at the end of period t, it must be that

$$b_t^1(y^t) + \delta U_{t+1}^1(y^t) \geq -\delta w_{t+1}^2(y^t)$$

for agent 1 not to make the above deviation. By Lemma A4, we already know that $U_{t+1}^2 = 0$ and $w_{t+1}^2 = -u$. Using this fact and the above inequality, we obtain the desired result. Q.E.D.

Given y^{t-1}, define the *incentive value* for agent 1 as follows:

$$\Phi(y^{t-1}) \equiv b_t^1(Y, y^{t-1}) + \delta U_{t+1}^1(Y, y^{t-1}) - \{b_t^1(0, y^{t-1}) + \delta U_{t+1}^1(0, y^{t-1})\}.$$

By Lemma A4 and IC_1, we obtain

$$\Phi_t(y^{t-1}) \geq c/\Delta p + b_t^2(Y) - b_t^2(0) = (2c)/\Delta p$$

for each y^{t-1} so that

$$\Phi_t(y^{t-1}) \geq (2c)/\Delta p. \tag{A9}$$

In addition, we have $\delta V_{t+1}(Y, y^{t-1}) \geq b_t^1(Y, y^{t-1})$ from DEP. Then, since $U_{t+1}^2(Y, y^{t-1}) = 0$ by Lemma A4, we have

$$\delta\{V_{t+1}(Y, y^{t-1}) + U_{t+1}^1(Y, y^{t-1})\} = \delta S^* \geq b_t^1(Y, y^{t-1}) + \delta U_{t+1}^1(Y, y^{t-1}).$$

Since $b_t^1(0, y^{t-1}) + \delta U_{t+1}^1(0, y^{t-1}) \geq \delta u$ by Lemma A4 and A5, this yields

$$\begin{aligned}
\delta S^* &\geq b_t^1(Y, y^{t-1}) + \delta U_{t+1}^1(Y, y^{t-1}) \\
&\geq b_t^1(Y, y^{t-1}) + \delta U_{t+1}^1(Y, y^{t-1}) - \{b_t^1(0, y^{t-1}) + \delta U_{t+1}^1(0, y^{t-1}) - \delta u\} \\
&= \Phi_t(y^{t-1}) + \delta u
\end{aligned}$$

so that

$$\delta S^* - \delta u \geq \Phi_t(y^{t-1}). \tag{A10}$$

From (A9) and (A10), we obtain condition (6) in Proposition 2.

Next, we show that condition (7) in Proposition 2 must be satisfied to ensure that agent 1 cannot make a profitable deviation against agent 2. For agent 1 not to deviate from b_t^2 while honoring to pay b_t^1, we need

$$-b_t^2(y^t) + \delta U_{t+1}^1(y^t) \geq \delta D_{t+1}^{12}(y^t) \tag{A11}$$

where $D_{t+1}^{12}(y^t)$ on the RHS denotes the continuation value agent 1 can obtain from the deviation against agent 2 in period t. Let $\tilde{U}_{t+1}^1(y^t)$ denote a lower bound for the continuation value agent 1 can secure after the deviation, hence $D_{t+1}^{12}(y_t) \geq \tilde{U}_{t+1}^1(y_t)$. We can define $\tilde{U}_{t+1}^1(y^t)$ as follows: For each $s \geq t+1$,

$$\tilde{U}_s^1(y^{s-1}) \;=\; w_s^1(y^{s-1}) + \max_{a_{1,s}\in\{0,1\}} \{E[\max\{b_s^1(y^s) + \delta\tilde{U}_{s+1}^1(y^s), 0\}|a_{1,s}, 0] - c(a_{1,s})\}.$$

The above $\tilde{U}_s^1(y^{s-1})$ is obtained since agent 1 continues to maintain the equilibrium relational contract with the principal after the deviation and agent 2 chooses $a_{2,s} = 0$ for all $s \geq t+1$. Agent 1 can always secure the reservation payoff of zero by choosing low effort and making no transfers to the principal, obtaining the payoff $\max\{b_s^1(y^s) + \delta\tilde{U}_{s+1}^1(y^s), 0\}$ in each period after his deviation against agent 2.

Let $\Delta q(a) \equiv p(1,1) - p(a,0)$ and $q(a) \equiv p(a,0)$. Define the *continuation bonus* for agent 1 from period t given $y_t = 0$ as follows:

$$\phi_t(y^{t-1}) \equiv b_t^1(0, y^{t-1}) + \delta U_{t+1}^1(0, y^{t-1}).$$

Note that $\phi_t(y^{t-1}) \geq \delta u$ due to Lemma A5. Then, we can rewrite $\tilde{U}_t^1(y^{t-1})$ in terms of $\phi_t(y^{t-1})$ and the incentive value $\Phi_t(y^{t-1})$ as follows:

$$\begin{aligned}
\tilde{U}_t^1(y^{t-1}) \;&=\; w_t^1 + \max_a\{E[\max\{b_t^1(y^t) + \delta\tilde{U}_{t+1}^1(y^t), 0\}|a, 0] - c(a)\} \\
&=\; U_t^1(y^{t-1}) - E[b_t^1(y^t) + \delta U_{t+1}^1(y^t)|1, 1] + c \\
&\quad + \max_a\{E[\max\{b_t^1(y^t) + \delta\tilde{U}_{t+1}^1(y^t), 0\}|a, 0] - c(a)\} \\
&=\; U_t^1(y^{t-1}) + c - E[b_t^1(y^t) + \delta U_{t+1}^1(y^t)|1, 1] \\
&\quad + \max_a\{E[\max\{b_t^1(y^t) + \delta U_{t+1}^1(y^t) + \delta\{\tilde{U}_{t+1}^1(y^t) - U_{t+1}^1(y^t)\}, 0\}|a, 0] - c(a)\} \\
&=\; U_t^1(y^{t-1}) + c - p(1,1)\Phi_t(y^{t-1}) - \phi_t(y^{t-1}) \\
&\quad + \max_a \Big\{q(a)\max\{\Phi_t(y^{t-1}) + \phi_t(y^{t-1}) + \delta\{\tilde{U}_{t+1}^1(Y, y^{t-1}) - U_{t+1}^1(Y, y^{t-1})\}, 0\} \\
&\quad + (1 - q(a))\max\{\phi_t(y^{t-1}) + \delta\{\tilde{U}_{t+1}^1(0, y^{t-1}) - U_{t+1}^1(0, y^{t-1})\}, 0\} - c(a)\Big\}.
\end{aligned}$$

By defining the net deviation gain as $\Gamma_t(y^{t-1}) \equiv \tilde{U}_t^1(y^{t-1}) - U_t^1(y^{t-1})$, we obtain

$$\begin{aligned}
\Gamma_t(y^{t-1}) \;&=\; c - p(1,1)\Phi_t(y^{t-1}) - \phi_t(y^{t-1}) \\
&\quad + \max_a \;\{q(a)\max\{\Phi_t(y^{t-1}) + \phi_t(y^{t-1})\delta\Gamma_{t+1}(Y, y^{t-1}), 0\} \\
&\quad + (1 - q(a))\max\{\phi_t(y^{t-1}) + \delta\Gamma_{t+1}(0, y^{t-1}), 0\} - c(a)\}.
\end{aligned}$$

Consider now the following problem.

Problem M. Given $y_1 \in \{0, Y\}$,

$$\min_{\{\Phi_t, \phi_t\}_{t=2}^{\infty}} \Gamma_2(y_1)$$

subject to (A9), (A10) and the dynamic enforcement constraint expressed as

$$\delta S^* \geq \Phi_t(y^{t-1}) + \phi_t(y^{t-1}), \quad \text{for any } t \geq 2 \tag{A12}$$

In the above, (A12) follows from DEP, $\delta V_t(Y, y^{t-2}) \geq b^1_{t-1}(Y, y^{t-2})$, and the fact that $U^1_t(Y, y^{t-2}) \geq 0$ and $V_t(Y, y^{t-2}) + U^1_t(Y, y^{t-2}) = S^*$. Note first that (A12) must be binding at the solution to Problem M because $\Gamma_t(y^{t-1})$ can be reduced by increasing $\phi_t(y^{t-1})$. Substituting $\delta S^* = \Phi_t(y^{t-1}) + \phi_t(y^{t-1})$ into $\Gamma_t(y^{t-1})$, we obtain

$$\begin{aligned}
\Gamma_t(y^{t-1}) \quad = \quad & c - \delta S^* + (1 - p(1,1))\Phi_t(y^{t-1}) \\
& + \max_a \big\{q(a) \max\{\delta S^* + \delta\Gamma_{t+1}(Y, y^{t-1}), 0\} \\
& + (1 - q(a)) \max\{\delta S^* - \Phi_t(y^{t-1}) + \delta\Gamma_{t+1}(0, y^{t-1}), 0\} - c(a)\big\}.
\end{aligned}$$

Neither $\Phi_{t-1}(y^{t-2})$ nor y^{t-1} affects the current value of $\Gamma_t(y^{t-1})$. Thus, the value of $\Phi_t(y^{t-1})$ that solves Problem M is independent of t and y^{t-1}, which we denote by Φ^*. Then the minimized value of $\Gamma_t(y^{t-1})$ is also independent of t and y^{t-1}. We denote this by Γ^*, which satisfies

$$\begin{aligned}
\Gamma^* \quad = \quad & \min_{\Phi \in [2c/\Delta p, \delta S^* - \delta u]} \Big\{c - \delta S^* + (1 - p(1,1))\Phi + \max_{a \in \{0,1\}} \big\{q(a) \max\{\delta S^* + \delta\Gamma^*, 0\} \\
& + (1 - q(a)) \max\{\delta S^* - \Phi + \delta\Gamma^*, 0\} - c(a)\big\}\Big\}.
\end{aligned}$$

This implies that $-b^2(Y) = -c/\Delta p \geq \delta\Gamma_2(Y) \geq \delta\Gamma^*$, resulting in condition (7) of Proposition 2. If this is not satisfied, we can find $\{\Phi_t(y^{t-1}), \phi_t(y^{t-1})\}_{t=1}^{\infty}$ such that (A9), (A10) and (A11) are all satisfied but $-b^2(Y) < \delta\Gamma^* \leq \delta\Gamma_2(Y)$ so that $-b^2(Y) < \delta\Gamma_2(Y)$, which violates agent 1's DEA in $t = 1$. Thus condition (7) in Proposition 2 is necessary. This completes the proof of the necessity part.

Sufficiency: See the Online Appendix.

7.3 Proof of Proposition 3

Define $x \equiv \delta/(1 - \delta) \in [0, \infty)$ and $\alpha(x) \equiv c/\Delta p + \pi x$. Then condition (2) in Proposition 1 can be rewritten as $s^* x \geq G(x) \equiv \max\{2c/\Delta p, \alpha(x)\}$. Suppose first $\Delta g Y \leq 2c$. Then $\Delta p Y < \Delta g Y \leq 2c$ by Assumption 2, hence $s^* x < G(x)$ for all $x \geq 0$. Thus we have $\delta_C(\gamma) = 1$ for all $\gamma \in [0, 1]$. Suppose next $\Delta g Y > 2c$, Then there exists a unique $\gamma^* \in [0, 1]$ such that $s^* = \pi$, hence $\delta_C(\gamma^*) = 1$ and $\delta_C(\gamma) < 1$ for all $\gamma < \gamma^*$. By continuity, we have $\delta_C(\gamma) > \delta_H(\gamma)$ for all $\gamma \in [0,1]$ as long as Δg is not too large. Note that $s^*x = G(x) = 2c/\Delta p$ holds at $x = 2c/s^*\Delta p$ when $(\Delta p - p(0,1))Y \geq 2c$ holds. Here, $(\Delta p - p(0,1))Y \geq 2c$ is equivalent to $\gamma \leq \hat{\gamma}$ for some $\hat{\gamma} \in [0,1]$, given $\Delta g \geq g(0,1) + 2c/Y$. In this case we can find some $x_c \geq 0$ such that $s^*x_c = 2c/\Delta p$ holds for any $\gamma \in [0,\hat{\gamma}]$. By letting $\delta_C/(1-\delta_C) \equiv x_c$, we then have $\delta_C(\gamma) < \delta_H(\gamma)$ for any $\gamma \in [0,\hat{\gamma}]$ because the first best is never attained under hierarchy at $x = 2c/s^*$: $\delta S^* = 2c/\Delta p < \delta u + 2c/\Delta p$ at $x = 2c/s^*\Delta p$. Then, by continuity, there exists some $\overline{\Delta g} > 2c/Y$, where $\overline{\Delta g} < 2c/Y + g(0,1)$, such that $\delta_H(\gamma) > \delta_C(\gamma)$ for all $\gamma \in [0,\tilde{\gamma}]$ for some $\tilde{\gamma} < \gamma^*$, given $\Delta g > \overline{\Delta g}$. Q.E.D.

Supplementary Material (Not for Publication) on "Relational Contracts and Hierarchy"

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In this supplementary material, we provide the proofs omitted in the paper and discuss the case where all decisions except effort choice are publicly observable.

8 Omitted Proofs

8.1 Proof of the sufficiency part in Proposition 1

Suppose the conditions in Proposition 1 are satisfied. First, we show that these conditions imply

$$\delta S^* \geq \max\left\{2c/\Delta p, c/\Delta p + \frac{\delta}{1-\delta}\pi, c/\Delta p + \frac{\delta}{1-\delta(1-p(1,0))}\tilde{\pi}\right\} \tag{S1}$$

where $\tilde{\pi} \equiv p(1,0)Y + u$ and $u \equiv p(1,0)c/\Delta p$. To see this, let $x \equiv \delta/(1-\delta) \in [0,\infty)$, and define

$$\alpha(x) \equiv c/\Delta p + \pi x$$

and

$$\beta(x) \equiv c/\Delta p + \frac{x}{1+p(1,0)x}\tilde{\pi}.$$

We will show that the conditions in Proposition 1 imply $s^*x \geq \beta(x)$, which then implies (S1). To show $s^*x \geq \beta(x)$, note that $\alpha(0) = \beta(0) = c/\Delta p$ and $\alpha(x) \geq \beta(x)$ holds if and only if $x \geq \tilde{x} \equiv (c/\Delta p)/p(0,1)Y$. Also $\alpha(\tilde{x}) = \beta(\tilde{x}) = 2c/\Delta p$. Thus $2c/\Delta p \geq \max\{\alpha(x), \beta(x)\}$ for all $x \leq \tilde{x}$ and $\alpha(x) \geq \max\{2c/\Delta p, \beta(x)\}$ for all $x \geq \tilde{x}$. Thus $s^*x \geq \max\{2c/\Delta p, \alpha(x)\}$ implies $s^*x \geq \beta(x)$.

Next, consider the following stationary contracts: in each period, the principal pays $\hat{b}^i_p(Y) = c/\Delta p$, $\hat{b}^i_p(0) = 0$ and $\hat{w}^i_p = 0$ to agent i, and agent i pays the principal $\hat{b}^p_i(y) = 0$ for any $y \in \{Y, 0\}$ and $\hat{w}^p_i = u \equiv p(1,1)c/\Delta p - c > 0$. Thus $\hat{b}^i(Y) \equiv \hat{b}^i_p(Y) - \hat{b}^p_i(Y) = c/\Delta p$ and $\hat{w}^i \equiv \hat{w}^i_p - \hat{w}^p_i = -u$ in every period. Then, agent i obtains equilibrium value of $\hat{U}^i = 0$ and the principal obtains equilibrium value $V = S^*$ equal to the discounted present value of the first-best surplus. Given the above contracts, consider now the following strategies.

The principal's strategies:

(i) Suppose that the principal and agent i make payments following the stationary contract stipulated above in all periods until the end of period $t-1$. Then, in the beginning of period t, the principal makes an up-front payment $w^i_{p,t} = \hat{w}^i_p = 0$.

(ii) Suppose that the principal and agent i make payments following the stationary contract stipulated above in all periods until the beginning of period t, hence $w^i_t =$

$\hat{w}^i$. Then, in period t, the principal does not exercise the default option against agent i and makes a bonus payment $b^i_{p,t}(y_t) = \hat{b}^i_p(y_t)$ for $y_t \in \{Y, 0\}$, as stipulated in the stationary contract.

(iii) Suppose the principal and agent i follow the stipulated stationary contract until the beginning of period t that includes the up-front payment, but agent i (or the principal) deviates by choosing $b^p_{i,t}(y_t) \neq \hat{b}^p_i(y_t) = 0$ (or $b^i_{p,t}(y_t) \neq \hat{b}^i_p(y_t)$) for some y_t at the end of period t. Then the principal punishes agent i in all future periods, by choosing $w^i_{p,s} = 0$ and exercising the default option against agent i for all $s \geq t+1$. For agent $j \neq i$, the principal does not exercise the default option and chooses either of the following, where $V^* \equiv \pi/(1-\delta)$ is the discounted present value (hereafter, continuation value) of the principal's payoffs obtained after she made a deviation against one agent.

i) If $\delta V^* \geq c/\Delta p$, then the principal pays bonus $b^j_{p,s} = \hat{b}^j_p(y_s)$ for all $s \geq t+1$;

ii) If $\delta V^* < c/\Delta p$, then the principal pays bonus $b^j_{p,s}(y_s) = 0$ for all y_s and $s \geq t+1$.

(iv) Suppose the principal and agent i follow the stipulated stationary contract until period $t-1$ but, in the beginning of period t, agent i (or the principal) deviates by choosing $w^p_{i,t} \neq \hat{w}^p_i$ (or $w^i_{p,t} \neq \hat{w}^i_p$). Following this, the principal exercises the default option against agent i in all periods $s \geq t$.

Agent i's Strategy:

(i) Suppose the principal and agent i follow the stipulated stationary contract until period $t-1$. In the beginning of period t, agent i makes the stipulated up-front payment $w^p_{i,t} = \hat{w}^p_i = u$.

(ii) In addition to (i), suppose $w^i_t = \hat{w}^i$ and the principal did not exercise the default option. Then agent i chooses $a_{i,t} = 1$.

(iii) Following (ii), agent i pays $b^p_{i,t}(y_t) = 0$ for any $y_t \in \{Y, 0\}$.

(iv) Suppose the principal and agent i follow the stipulated stationary contract until the beginning of period t that includes the up-front payment, but agent i (or the principal) deviates by choosing $b^p_{i,t}(y_t) \neq \hat{b}^p_i(y_t) = 0$ (or $b^i_{p,t}(y_t) \neq \hat{b}^i_p(y_t)$) for some y_t at the end of period t. Then agent i pays $w^p_{i,s} = 0$ to the principal in all periods $s \geq t+1$.

(v) Suppose the principal and agent i follow the stipulated stationary contract until period $t-1$ but, in the beginning of period t, agent i (or the principal) deviates by choosing $w^p_{i,t} \neq \hat{w}^p_i$ (or $w^i_{p,t} \neq \hat{w}^i_p$). Following this, agent i chooses $a_{i,s} = 0$ in all periods $s \geq t$.

(vi) If the principal exercises the default option against agent i in some period t, then agent i chooses $a_{i,s} = 0$ for any $s \geq t$.

We show below that the strategies described above constitute an equilibrium if the conditions in Proposition 1 are satisfied. In what follows, we refer to them as equilibrium when there is no confusion.

First, we show that agent i does not benefit from unilateral deviation. If all players follow the above strategies, then agent i obtains the equilibrium payoff $\hat{U}_t^i = 0$ in each period t. We further observe the following. First, agent i chooses high effort $a_{i,t} = 1$ on the equilibrium path because of IC: $p(1,1)\hat{b}^i(Y) - c \geq p(0,1)\hat{b}^i(Y)$, which holds as equality. Second, if agent i pays $w_{i,t}^p \neq \hat{w}^i$ in the beginning of some period t despite the equilibrium play by the principal and agent i in the past, then the principal will exercise the default option against agent i from period t onward. Given this, agent i chooses $a_{i,t} = 0$ from period t forever. Third, if agent i deviates by choosing $b_{i,t}^p(y_t) \neq \hat{b}_i^p(y_t) = 0$ for some y_t in some period t despite the equilibrium play by the principal and agent i in the past, then agent i expects the principal to exercise the default option after choosing $w_{p,s}^i = 0$ for all $s \geq t+1$. Thus agent i expects to obtain the continuation payoff of zero from period $t+1$. The largest possible deviation payoff agent i can expect from the end of period t is then $-b_{i,t}^p(y_t) + \delta \times 0 \leq 0$. Since the equilibrium value from period t is $-\hat{b}_i^p(y_t) + \delta\hat{U}^i = 0$, agent i never makes such a deviation. We can also verify that agent i has no incentives to deviate from $\hat{w}_i^p = u$, given the principal's equilibrium strategy. If agent i pays $w_{i,t}^p \neq \hat{w}_{i,t}^p = u$, then the principal exercises the default option against agent i immediately following the deviation. This gives agent i at most the payoff of $-w_{i,t}^p + \hat{w}_p^i = -w_{i,t}^p \leq 0$, which is smaller than the equilibrium value $\hat{U}^i = 0$.

Next, we show that the principal does not benefit from any unilateral deviation. Note that the principal expects to earn the discounted present value S^* from the beginning of each period on the equilibrium path.

Case 1: Suppose neither agent deviated until period t but the principal deviates against both agents by choosing $b_p^1(y_t) = b_p^2(y_t) = 0$ when $y_t = Y$. This saves the principal $2c/\Delta p$ in period t, but both agents choose low effort in all future periods, leading to zero continuation value for the principal. Since $\delta S^* \geq 2c/\Delta p$, the principal does not benefit from the deviation.

Case 2: Suppose neither agent deviated until period t but the principal deviates against only one agent, say agent i, by choosing $b_p^i(y_t) = 0$ when $y_t = Y$. Then agent i chooses low effort and the principal exercises the default option in all future periods. Thus, the principal interacts only with agent $j \neq i$ from period $t+1$ onward. There are four cases to consider for the continuation game from period $t+1$.

Case 2-1: The principal maintains equilibrium relational contract with agent j forever, which leads to the continuation value V^*. Then, by the afore-mentioned deviation against agent i only, the principal gains $\hat{b}^i(Y) + \delta V^*$. But this is not larger that the equilibrium continuation value since $\delta S^* \geq \delta V^* + c/\Delta p$.

Case 2-2: The principal pays $w^j_{p,t+1} \neq \hat{w}^j_p = 0$. Following this, agent j chooses low effort from period $t+1$, which gives the principal the continuation value of $-w^j_{p,t+1} + \hat{w}^p_j$ from agent j in period $t+1$. Note that $-w^j_{p,t+1} + \hat{w}^p_j \leq \hat{w}^p_j = -\hat{w}^j = u$ and that $\tilde{\pi} = p(1,0)Y + u \geq u$. Then we have $\delta S^* \geq \hat{b}^i(Y) + \delta\tilde{V} \geq \hat{b}^i(Y) + \delta\tilde{\pi} \geq \hat{b}^i(Y) + \delta u$, which shows that the above deviation is not profitable.

Case 2-3: If the principal deviates against agent j by exercising the default option in period $t+1$, then her continuation value is zero. If the principal does not deviate against agent j, then agent j still has the same belief as that on the equilibrium path. Thus agent j chooses $a_{j,s} = 1$ for all $s \geq t+1$. By not deviating against agent j, the principal can secure a positive continuation value $V^* = \pi + \delta V^* > 0$. Thus, the principal does not exercise the default option against agent j.

Case 2-4: The principal pays $\hat{w}^j_p = 0$ but chooses $b^j(Y) \neq \hat{b}^j(Y) = c/\Delta p$ while choosing $\hat{b}^j(0) = 0$ in period $s \geq t+1$. Following this, agent j chooses $a_j = 0$ forever, which gives the principal the continuation value from period $t+1$ as follows:

$$\tilde{V} = \tilde{\pi} + \delta(1 - p(1,0))\tilde{V}$$

where $\pi \equiv p(1,0)Y$. But then, the principal's assumed deviation against agent i along with the above deviation against agent j gives the principal a smaller payoff than her equilibrium payoff. It is because the principal's deviation gain is $\hat{b}^i(Y) + \delta\tilde{V}$ but her discounted equilibrium value is $\delta S^* \geq c/\Delta p + \delta\tilde{V}$.

Finally, we check that the principal chooses her best responses off the equilibrium path after the deviation $b^i_t \neq \hat{b}^i(y_t)$ against agent i. If the principal deviates against agent $j \neq i$ as well, that is, $b^j_t \neq \hat{b}^j$, then both agents choose zero transfer and low effort from the next period onward. Given this, the principal will optimally exercise the default option against both agents from the next period. This leaves the case where $b^j_t = \hat{b}^j$, which we consider below.

First, suppose that $\delta V^* \geq c/\Delta p$. (i) Suppose that the principal deviates by choosing $b^j_{p,s}(Y) = 0$ in some period $s \geq t+1$. Then agent j chooses low effort from period $s+1$ forever, leading to the principal's deviation gain $\hat{b}^j(Y) + \delta \times 0 = c/\Delta p$. But this is smaller than δV^*, which the principal can attain by not deviating against agent j in period s. (ii) Suppose the principal deviates by choosing $w^j_p \neq \hat{w}^j_p$ in some period $s \geq t+1$. This leads to her deviation gain $-w^j_p + \hat{w}^p_j \leq -\hat{w}^j = u$ because agent j will choose low effort forever following the deviation. But $\delta V^* \geq c/\Delta p$ implies $V^* \geq c/\Delta p > p(1,1)c/\Delta p - c = u$, hence the principal has no incentive to make such a deviation. (iii)

Suppose the principal deviates by exercising the default option against agent j in some period $s \geq t+1$ after paying the stipulated up-front transfer $\hat{w}_p^j = 0$. Then the principal will earn the continuation value of zero; by not exercising the default option, she can earn the continuation value $p(1,0)Y - E[\hat{b}^j(y)|1,0] + \delta V^* = V^* - u > 0$.

Second, suppose that $\delta V^* < c/\Delta p$. This implies $\delta \tilde{V} < c/\Delta p$.[12] Then, it is optimal for the principal to pay $b_s^j(Y) = 0$ rather than the stipulated informal bonus $\hat{b}^j(Y) = c/\Delta p$ for any $s \geq t+1$. To see this, suppose the principal makes a one-off deviation from the specified strategy by choosing $b_s^j(Y) \neq 0$ and $b_\tau^j(Y) = 0$ for all $\tau \geq s+1$. If $b_s^j(Y) \neq c/\Delta p$, then agent j chooses low effort forever, giving the principal payoff of zero. This is not profitable. If $b_s^j(Y) = c/\Delta p$, then the principal obtains $-b_s^j(Y) + \delta \tilde{V} = -c/\Delta p + \delta \tilde{V} < 0$, which is not profitable either. Thus, the principal pays $b_{p,s}^j(Y) = 0$ (which occurs with probability $p(1,0)$) and $b_{p,s}^j(0) = 0$ (which occurs with probability $1 - p(1,0)$) in period $s \geq t+1$, leading to the continuation value $\tilde{V}$.

Additionally, if the principal deviates by choosing $w_p^j \neq \hat{w}_p^j = 0$ in period $t+1$, then she obtains $-w_p^j + \hat{w}_j^p \leq -\hat{w}^j = u$. But we have $-\hat{w}^j = u \leq \tilde{\pi} < \tilde{V}$. Also, if the principal makes the stipulated up-front transfer $\hat{w}_p^j = 0$ but exercises the default option against agent j in some period $s \geq t+1$, then she will earn $-\hat{w}^j + 0 = u$ in period s, which is again less than $\tilde{V}$. Thus, the principal does not deviate from $\hat{w}_p^j$ and does not exercise the default option against agent j in period $t+1$ when $y_t = 0$, whereas she chooses $b_{p,t+1}^j(Y) = 0$ when $y_{t+1} = Y$ (but she will follow the stipulated informal bonus $\hat{b}^j(0) = 0$ when $y_{t+1} = 0$). This gives her the continuation value of $\tilde{V}$ in period $t+1$. Q.E.D.

8.2 Proof of Lemma A4

Let $\hat{b}_t^i(y^t)$ and $\hat{w}_t^i(y^{t-1})$ be the net informal bonus and net up-front transfer for agent i in the original equilibrium, which attains the first best surplus S^*, given the output history $y^t = (y_1, ..., y_t)$ and $y^{t-1} = (y_1, ..., y_{t-1})$. Let $\hat{U}_t^i$ denote the equilibrium discounted present value (hereafter, equilibrium value) of agent i's payoffs at the beginning of period t. Consider now the alternative transfers for agent 2 $\{\tilde{w}_t^2, \tilde{b}_t^2\}_{t=1}^{\infty}$ defined as follows: (i) $\tilde{b}_t^2(y^t) = \tilde{b}^2(y_t) \equiv c/\Delta p$ for $y_t = Y$ and $\tilde{b}_t^2(y^t) = \tilde{b}^2(y_t) \equiv 0$ for $y_t = 0$, and (ii) $\tilde{w}_t^2(y^{t-1}) = \tilde{w}^2 \equiv -u$ regardless of y^{t-1}. Define also $\tilde{b}_{1,t}^2 = \tilde{b}^2$ and $\tilde{b}_{2,t}^1 = 0$, and $\tilde{w}_{1,t}^2 = 0$ and $\tilde{w}_{2,t}^1 = -\tilde{w}^2$. Based on the newly defined transfers, we can construct a new equilibrium that attains the first best surplus S^* as well. Thus, without loss of generality, we can focus on these transfers.

In the following, we first specify strategies chosen by each player. We then show that they constitute an equilibrium that attains the first-best surplus S^*.

Strategy profile (*):

The principal's strategy:

[12] This follows from the fact that V satisfying $V = p(1,0)Y + u + p(1,0)\max\{-c/\Delta p + \delta V, 0\} + (1 - p(1,0))\delta V$ yields $V = V^*$ if $\delta V^* \geq c/\Delta p$ and $V = \tilde{V}$ if $\delta V^* < c/\Delta p$. In the latter case we have $\delta \tilde{V} < c/\Delta p$.

- The principal pays the stipulated up-front transfer $\hat{w}^1_{p,t}(y^{t-1})$, does not exercise the default option and then pays informal bonus $\hat{b}^1_{p,t}(y^t)$ to agent 1 as long as she and agent 1 have followed the stipulated payments in the past before period t.

- The principal pays $w^1_{p,t} = 0$ to agent 1 and exercises the default option against agent 1 once $b^1_s \neq \hat{b}^1_s(y^s)$ or $w^1_s \neq \hat{w}^1_s(y^{s-1})$ in some past period $s \leq t-1$. Also, the principal pays $b^1_{p,t} = 0$ to agent 1 once $w^1_s \neq \hat{w}^1_s(y^{s-1})$ or $b^1_{s-1} \neq \hat{b}^1_{s-1}(y^{s-1})$ in some past or current period $s \leq t$.

Agent 1's strategy:

- Agent 1 pays $\hat{w}^p_{1,t}(y^{t-1})$ and $\hat{b}^p_{1,t}(y^t)$ to the principal and $\tilde{w}^2_1$ and $\tilde{b}^2_1(y_t)$ to agent 2. In addition, agent 1 does not exercise the default option against agent 2 and chooses $a_{1,t} = 1$ as long as all the players have followed the stipulated payments in the past before period t.

- Agent 1 pays $w^p_{1,t} = w^2_{1,t} = 0$, chooses $a_{1,t} = 0$, and exercises the default option against agent 2 if both of (i) and (ii) occur: (i) $b^1_s \neq \hat{b}^1_s(y^s)$ or $w^1_s \neq \hat{w}^1_s(y^{s-1})$ for some $s \leq t-1$; (ii) $b^2_s \neq \tilde{b}^2(y_s)$ or $w^2_s \neq \tilde{w}^2$ for some $s \leq t-1$.

- Suppose that $b^2_s \neq \tilde{b}^2(y_s)$ for some $s \leq t-1$ while $b^1_s = \hat{b}^1_s(y^s)$ for all $s \leq t-1$. Then agent 1 pays $w^2_{1,t} = b^2_{1,t} = 0$ and exercises the default option against agent 2. Also, agent 1 makes the stipulated up-front transfer $\hat{w}^p_{1,t}$ to the principal if $U^{1*}_t(y^{t-1}) > \hat{w}^1_{p,t}(y^{t-1})$ where $U^{1*}_t(y^{t-1})$ is the discounted present value of his payoffs, defined as

$$U^{1*}_t(y^{t-1}) = \max\left\{\hat{w}^1_{p,t}(y^{t-1}), \hat{w}^1_t(y^{t-1}) + \max_{a_{1,t}\in\{0,1\}} E_{y_t}[\max\{\hat{b}^1_t(y^t) + \delta U^{1*}_{t+1}(y^t), 0\}|a_{1,t}, 0] - c(a_{1,t})\right\}.$$

  If $U^{1*}_t(y^{t-1}) \leq \hat{w}^1_{p,t}(y^{t-1})$, then agent 1 pays $w^p_{1,t} = 0$ to the principal and chooses $a_{1,t} = 0$ in period t.

- If $w^2_s \neq \tilde{w}^2$ for some $s \leq t$ while $w^1_s = \hat{w}^1_s(y^{s-1})$ for all $s \leq t$, then agent 1 pays the stipulated informal bonus $b^2_{1,t} = 0$ to agent 2 and exercises the default option against him in period t. In addition, agent 1 makes payment of $\hat{b}^p_{1,t}(y^t)$ to the principal if $\hat{b}^1_t(y^t) + \delta U^{1*}_{t+1}(y^t) \geq \hat{b}^1_{p,t}(y^t)$ and $b^p_{1,t} = 0$, otherwise.

Agent 2's strategy:

- Agent 2 pays $\tilde{b}^1_2$ and $\tilde{w}^1_2$ to agent 1, and chooses $a_{2,t} = 1$ if both agents have followed the stipulated payments in the past before period t.

- Agent 2 pays $\tilde{w}^1_2 = u$ to agent 1 in period t if $\{\tilde{w}^2, \tilde{b}^2\}$ has been paid for all $s \leq t-1$. Otherwise agent 2 pays $w^1_{2,t} = 0$ and chooses $a_{2,t} = 0$ in period t.

- Agent 2 chooses $a_{2,t} = 1$ and pays $\tilde{b}^1_2(y_t)$ to agent 1 if $\{\tilde{w}^2, \tilde{b}^2\}$ has been paid for all $s \leq t$. Otherwise, agent 2 chooses $a_{2,t} = 0$ and pays $b^1_{2,t} = 0$ to agent 1 in period t. More precisely, if $w^2_t \neq \tilde{w}^2$, then agent 2 chooses $a_{2,t} = 0$. Also, if $b^2_s \neq \tilde{b}^2$ for some $s \leq t-1$, then agent 2 pays $w^1_{2,s+1} = 0$ and chooses $a_{2,s+1} = 0$.

Now define

$$U_t^1(y^{t-1}) \equiv \sum_{i=1,2} \hat{U}_t^i(y^{t-1})$$

for $t \geq 1$. Note that agent 2 obtains no rent, i.e., $U_t^2(y^{t-1}) = 0$, under the newly defined transfers $\{\tilde{b}^2, \tilde{w}^2\}$. In what follows, we will suppress the argument y^{s-1} from $\hat{U}_{s+1}^i$ and write $\hat{U}_{s+1}^i(y_s, y^{s-1})$ simply as $\hat{U}_{s+1}^i(y_s)$.

Using agent 2's incentive compatibility (IC_2), we can re-write agent 1's incentive compatibility (IC_1) in the original equilibrium as

$$\begin{aligned}
&\hat{b}_t^1(Y) + \delta\hat{U}_{t+1}^1(Y) - \{\hat{b}_t^1(0) + \delta\hat{U}_{t+1}^1(0)\} \\
&\geq\ \hat{b}_t^2(Y) - \hat{b}_t^2(0) + c/\Delta p \\
&\geq\ -\delta\hat{U}_{t+1}^2(Y) + \delta\hat{U}_{t+1}^2(0) + 2c/\Delta p
\end{aligned}$$

so that

$$\hat{b}_t^1(Y) + \delta\sum_i \hat{U}_{t+1}^i(Y) - \left\{\hat{b}_t^1(0) + \delta\sum_i \hat{U}_{t+1}^i(0)\right\} \geq 2c/\Delta p. \tag{S2}$$

Then we have

$$\begin{aligned}
\hat{b}_t^1(Y) + \delta U_{t+1}^1(Y) - \{\hat{b}_t^1(0) + \delta U_{t+1}^1(0)\} \ &\geq\ c/\Delta p + c/\Delta p \\
&=\ \tilde{b}^2(Y) - \tilde{b}^2(0) + c/\Delta p,
\end{aligned}$$

which shows that (IC_1) is satisfied under the newly defined transfer scheme.

Turning to agent 1's dynamic enforcement constraint (DEA_1), we obtain the following condition when $y_t = Y$ is realized, given y^{t-1}:

$$\begin{aligned}
-\tilde{b}^2(Y) + \hat{b}_t^1(Y) + \delta U_{t+1}^1(Y) &= -\tilde{b}^2(Y) + \hat{b}_t^1(Y) + \delta\sum_i \hat{U}_{t+1}^i(Y) \\
&= -c/\Delta p + \hat{b}_t^1(Y) + \delta\sum_i \hat{U}_{t+1}^i(Y) \\
&\geq -\{\hat{b}_t^2(Y) + \delta\hat{U}_{t+1}^2(Y) - (\hat{b}_t^2(0) + \delta\hat{U}_{t+1}^2(0))\} \\
&\quad +\ \hat{b}_t^1(Y) + \delta\hat{U}_{t+1}^1(Y) + \delta\hat{U}_{t+1}^2(Y) \\
&= -\hat{b}_t^2(Y) + \hat{b}_t^1(Y) + \delta\hat{U}_{t+1}^1(Y) + \{\hat{b}_t^2(0) + \delta\hat{U}_{t+1}^2(0)\} \\
&\geq -\hat{b}_t^2(Y) + \hat{b}_t^1(Y) + \delta\hat{U}_{t+1}^1(Y) \\
&\geq \hat{b}_{p,t}^1(Y) + \delta \times 0.
\end{aligned} \tag{S3}$$

In the above, the first inequality follows from (IC_2) in the original equilibrium, the second inequality follows from (DEA_2) in the original equilibrium, i.e., $\hat{b}_t^2(0) + \delta\hat{U}_{t+1}^2(0) \geq 0$, and the last inequality is from the original (DEA_1), i.e., agent 1 can refuse payment to and from agent 2 while paying nothing to the principal (but receiving $\hat{b}_{p,t}^1$ from her) and choosing low effort from the next period forever. This must be not profitable:

$$-\hat{b}_t^2(Y) + \hat{b}_t^1(Y) + \delta\hat{U}_{t+1}^1(Y) \geq \hat{b}_{p,t}^1(Y). \tag{S4}$$

Then (S3) shows that

$$-\tilde{b}^2(Y) + \hat{b}^1_t(Y) + \delta U^1_{t+1}(Y) \geq \hat{b}^1_{p,t}(Y). \tag{S5}$$

Under the prescribed strategies (*), agent 1 would obtain the payoff on the RHS of (S5) if he deviates from both $\hat{b}^1_t(Y)$ and $\tilde{b}^2(Y) = c/\Delta p$. The deviation triggers punishment such that agent 1 receives no payment from the principal and agent 2 while agent 2 chooses low effort from the next period. Then, by (S5), it is not profitable for agent 1 to deviate from both $\hat{b}^1_t(Y)$ and $\tilde{b}^2_t(Y) = c/\Delta p$.

Next, consider the case where $y_t = 0$. Then

$$\begin{aligned} -\tilde{b}^2(0) + \hat{b}^1_t(0) + \delta U^1_{t+1}(0) &= \hat{b}^1_t(0) + \delta \sum_i \hat{U}^i_{t+1}(0) \\ &\geq \hat{b}^1_t(0) + \delta \hat{U}^1_{t+1}(0) \\ &\geq \hat{b}^1_{p,t}(0). \end{aligned}$$

In the above, the last inequality follows from the original (DEA_1) corresponding to $y_t = 0$: since agent 1 can refuse payment to the principal and choose low effort from the next period forever, we must have $\hat{b}^1_t(0) + \delta \hat{U}^1_{t+1}(0) \geq \hat{b}^1_{p,t}(0)$. Thus we obtain

$$-\tilde{b}^2_t(0) + \hat{b}^1_t(0) + \delta U^1_{t+1}(0) \geq \hat{b}^1_{p,t}(0). \tag{S6}$$

The prescribed strategies (*) then imply that agent 1 would obtain the payoff on the RHS of (S6) if he deviates from both $\tilde{b}^2(0) = 0$ and $\hat{b}^1_t(0)$. The deviation triggers punishment such that agent 1 receives no payment from the principal and agent 2, the principal exercises the default option against agent 1 from the next period, and agent 2 chooses low effort from the next period forever. Then, by (S6), it is not profitable for agent 1 to deviate from both $\tilde{b}^2(0) = 0$ and $\hat{b}^1_t(0)$.

Consider now the case where agent 1 deviates against agent 2 only. Recall that the principal cannot observes this. Suppose $b^1_t = \hat{b}^1_t(Y)$ but $b^2_t \neq \tilde{b}^2(Y)$. The fifth line in (S3) for $y_t = Y$ shows that

$$-\tilde{b}^2(Y) + \delta U^1_{t+1}(Y) \geq -\hat{b}^2_t(Y) + \delta \hat{U}^1_{t+1}(Y). \tag{S7}$$

The original (DEA_1) corresponding to $y_t = Y$ is given by

$$-\hat{b}^2_t(Y) + \delta \hat{U}^1_{t+1}(Y) \geq \delta D^{12}_{t+1}(Y) \tag{S8}$$

where $D^{12}_{t+1}(Y)$ is the continuation value that agent 1 obtains from the beginning of period $t+1$ if he deviates against only agent 2 in period t by choosing $b^1_t = \hat{b}^1_t(Y)$ but $b^2_t \neq \hat{b}^2_t(Y)$. Since agent 1 can always pay $w^2_{1,s} = b^2_{1,s} = 0$ to agent 2 and exercise the default option against him (thus $a_{2,s} = 0$ for all $s \geq t+1$) while maintaining the equilibrium play with the principal for all periods $s \geq t+1$, agent 1 can ensure at least

$$U^{1*}_s(y^{s-1}) = \max\left\{\hat{w}^1_{p,s}(y^{s-1}), \hat{w}^1_s(y^{s-1}) + \max_{a_{1,s}\in\{0,1\}} E[\max\{\hat{b}^1_s(y^2) + \delta \tilde{U}^1_{s+1}(y^s), 0\}|a_{1,s}, 0] - c(a_{1,s})\right\}$$

for $s \geq t+1$. In what follows, we use the shorthand notation $U^{1*}_{s+1}(y_s)$ for $U^{1*}_{s+1}(y_s, y^{s-1})$. Then $D^{12}_{t+1}(Y) \geq U^{1*}_{t+1}(Y)$ holds and, therefore, (S8) implies that

$$-\hat{b}^2_t(Y) + \delta\hat{U}^1_{t+1}(Y) \geq \delta D^{12}_{t+1}(Y) \geq \delta U^{1*}_{t+1}(Y)$$

so that

$$-\tilde{b}^2(Y) + \delta U^1_{t+1}(Y) \geq \delta U^{1*}_{t+1}(Y). \tag{S9}$$

Under the prescribed strategies (*), agent 1 obtains the continuation value $\delta U^{1*}_{t+1}(Y)$ by deviating against only agent 2 whereas he obtains the LHS of (S9) as his equilibrium value. Due to (S9), agent 1 cannot make profitable deviations from the constructed equilibrium strategy (*).

Similarly, when $y_t = 0$ is realized, we have

$$\begin{aligned} -\tilde{b}^2(0) + \delta U^1_{t+1}(0) &= \delta U^1_{t+1}(0) \\ &= \sum_i \delta\hat{U}^i_{t+1}(0) \\ &\geq \delta\hat{U}^1_{t+1}(0) - \hat{b}^2_t(0) \\ &\geq \delta D^{12}_{t+1}(0), \end{aligned}$$

where the first inequality follows from the original (DEA_2) corresponding to $y_t = 0$, i.e., $\hat{b}^2_t(0) + \delta\hat{U}^2_{t+1}(0) \geq 0$, and the last inequality is from (DEA_1), i.e., $-\hat{b}^2_t(0) + \delta\hat{U}^1_{t+1}(0) \geq \delta D^{12}_{t+1}(0)$. Again, since $D^{12}_{t+1}(0) \geq U^{1*}_{t+1}(0)$ holds, we have

$$-\tilde{b}^2(0) + \delta U^1_{t+1}(0) \geq \delta U^{1*}_{t+1}(0). \tag{S10}$$

Under the prescribed strategies (*), agent 1 obtains the continuation value given on the RHS of (S10) if he deviates against only agent 2 in period t. But, by (S10), such deviation is not profitable.

Let us now consider agent 1's deviation against only the principal. Note that we have

$$\begin{aligned} \hat{b}^1_t(y^t) + \delta U^1_{t+1}(y^t) &= \hat{b}^1_t(y^t) + \delta \sum_i \hat{U}^i_{t+1}(y^t) \\ &\geq \hat{b}^1_t(y^t) + \delta\hat{U}^1_{t+1}(y^t) \\ &\geq 0, \end{aligned} \tag{S11}$$

where the last inequality follows from the original (DEA_1): agent 1 can refuse payment to and from the principal, and choose low effort from the next period, which gives him the payoff of zero. We verify below that agent 1 will obtain the payoff of zero from period $t+1$ after such a deviation in period t. Under the prescribed strategy (*), if $b^1_t \neq \hat{b}^1_t(y^t)$, then the principal will pay $w^1_{p,t+1} = 0$ to agent 1 and exercise the default option against him from period $t+1$. Agent 1 will respond to this by choosing $w^p_{1,t+1} = 0$ and low effort. Also, if agent 1 still makes the stipulated transfers to agent 2, then he will obtain

$$-c(a_{1,s}) - \{E[\tilde{b}^2(y_s)|a_{1,s}, 1] + \tilde{w}^2\}$$

for each $s \geq t+1$. By the definition of $\tilde{b}^2$ and $\tilde{w}^2$, the above payoff becomes $-2c$ if $a_{1,s} = 1$ and $-c$ if $a_{1,s} = 0$. In either case, agent 1's payoff is negative. Thus agent 1 chooses to pay nothing to agent 2 and exercises the default option against him. This gives agent 1 the continuation value of zero from period $t+1$. Then (S11) implies that agent 1 never deviates from $\hat{b}^1_t(y_t)$.

Next, we show that agent 2's dynamic enforcement constraint (DEA_2) is satisfied as follows:

$$\tilde{b}^2(y^t) + \delta U^2_{t+1}(y^t) = \tilde{b}^2(y^t) \geq 0. \tag{S12}$$

If agent 2 deviates so that $b^2_t \neq \tilde{b}^2$, then he choose $w^1_{2,t+1} = 0$ and low effort from period $t+1$. Given this, agent 1 pays $w^2_{1,t+1} = b^2_{1,t+1} = 0$ and exercises the default option against agent 2 from period $t+1$. This gives agent 2 the continuation value of zero from period $t+1$.[13] Then, (S12) shows that agent 2 never deviates from $\tilde{b}^2(y_t)$.

We now check the principal's dynamic enforcement constraint (DEP). Denote by $\hat{V}_s$ the discounted present value of the principal's payoff in the original equilibrium from period s and let V_s be the discounted present value of her payoff from period s under the newly defined transfers. Then $V_s(y^{s-1}) = S^* - U^1_s(y^{s-1})$. In the original equilibrium, (DEP) implies that

$$\delta \hat{V}_{t+1}(y^t) \geq \hat{b}^1_t(y^t).$$

Note here that $V_{t+1}(y^t) = S^* - U^1_{t+1}(y^t) = S^* - \sum_i \hat{U}^i_{t+1}(y^t) = \hat{V}_{t+1}(y^t)$. If the principal makes a deviation such that $b^1_t \neq \hat{b}^1_t(y^t)$, then the prescribed strategies (*) lead to zero payoff for the principal payoff from period $t+1$ onward. Thus (DEP) is satisfied under the strategies (*) as constructed above: $\delta V_{t+1}(y^t) = \delta \hat{V}_{t+1}(y^t) \geq \hat{b}^1_t(y^t) + \delta \times 0$. In addition, the original (DEP) implies $\hat{V}_t(y^{t-1}) \geq \hat{w}^p_{1,t}$, that is, the principal does not deviate from paying $\hat{w}^1_{p,t} \geq 0$ while receiving only $\hat{w}^p_{1,t}$. Since $V_t(y^{t-1}) = \hat{V}_t(y^{t-1})$, we have $V_t(y^{t-1}) \geq \hat{w}^p_{1,t}$ under the strategies (*). Thus the principal never deviates from $\hat{w}^1_{p,t}$.

Put together, the strategies constructed in (*) constitute an equilibrium that attains the first-best surplus S^* in the beginning of period 1. Q.E.D.

8.3 Properties of Γ^*

We prove the two properties of Γ^* stated in the text. First, we show $\Gamma^* < 0$. Suppose to the contrary that $\Gamma^* \geq 0$. Then, since $\delta S^* \geq \delta S^* - \delta u \geq \Phi^*$, we have $\delta S^* - \Phi^* + \delta \Gamma^* \geq 0$. This implies that

$$\begin{aligned}\Gamma^* &= c - \delta S^* + (1 - p(1,1))\Phi^* + \max_{a \in \{0,1\}} \{q(a)\{\delta S^* + \delta \Gamma^*\} + (1 - q(a))\{\delta S^* - \Phi^* + \delta \Gamma^*\} - c(a)\} \\ &= c - \Delta q(\hat{a})\Phi^* - c(\hat{a}) + \delta \Gamma^*\end{aligned}$$

[13]Additionally, agent 1 will follow $\{\hat{b}^1_s, \hat{w}^1_s\}^{\infty}_{s=t+1}$ or pay nothing and choose low effort $a_{1,s} = 0$, depending on the value of $U^{1*}_s(y^s)$. If $U^{1*}_s > \hat{w}^1_{p,s}$, then agent 1 pays $\hat{w}^p_{1,s} \geq 0$ to the principal; otherwise, $U^{1*}_s = \hat{w}^1_{p,s}$ and hence agent 1 pays $w^p_{1,s} = 0$ and chooses low effort. Moreover, if $\hat{b}^1_s(y^s) + \delta U^{1*}_{s+1}(y^s) \geq \hat{b}^1_{p,s}(y^s)$, then agent 1 pays $\hat{b}^p_{1s}(y^s) \geq 0$ to the principal; otherwise, agent 1 pays $b^p_{1,s} = 0$, followed by paying $w^p_{1,s+1} = 0$ and choosing low effort in period $s+1$. The principal will follow $\{\hat{b}^1_s, \hat{w}^1_s\}$ if she and agent 1 have not deviated from these payments in the past. Otherwise, the principal will pay $w^1_{p,s+1} = 0$ and exercise the default option against agent 1 if $b^1_s \neq \hat{b}^1_s$ and/or she will pay $b^1_{p,s+1} = 0$ to agent 1 if $w^1_{s+1} \neq \hat{w}^1_{s+1}$.

where $\hat{a}$ is the optimal effort agent 1 chooses. This shows

$$(1-\delta)\Gamma^* = c - \Delta q(\hat{a})\Phi^* - c(\hat{a}).$$

If $\hat{a} = 1$, then the above expression is negative. If $\hat{a} = 0$, then the above expression becomes $c - \Delta q(0)\Phi^*$, which is also negative because $\Phi^* \geq 2c/\Delta p$ and $\Delta q(0) \equiv p(1,1) - p(0,0) \geq \Delta p = p(1,1) - p(1,0)$. This proves $\Gamma^* < 0$.

Next, we show that $\Gamma^* \to -\infty$ as $\delta \to 1$. First, claim $\Gamma^* + u < 0$ when $\delta \to 1$. To see this, suppose that $\Gamma^* + u \geq 0$ when $\delta \to 1$. Then we have $\Gamma^* + S^* \geq 0$ because $S^* \geq u$ as $\delta \to 1$. Since it becomes feasible to choose $\Phi = \delta S^* - \delta u$ in Problem M, we obtain

$$\begin{aligned}
\Gamma^* &\leq c - \delta S^* + (1-p(1,1))\{\delta S^* - \delta u\} + \max_a \{q(a)\delta S^* + \delta\Gamma^* + (1-q(a))\{\delta u + \delta\Gamma^*\} - c(a)\} \\
&= c - \Delta q(\hat{a})\{\delta S^* - \delta u\} - c(\hat{a}) + \delta\Gamma^*,
\end{aligned}$$

which leads to

$$(1-\delta)\Gamma^* \leq c - \Delta q(\hat{a})\{\delta S^* - \delta u\} - c(\hat{a}).$$

However, as $\delta \to 1$, the RHS of the above inequality approaches $-\infty$ because $S^* = s^*/(1-\delta) \to \infty$ as $\delta \to 1$. Thus we have $\Gamma^* \to -\infty$ as $\delta \to 1$. Thus $\Gamma^* + u < 0$ when $\delta \to 1$. Also, if $S^* + \Gamma^* < 0$ when δ is close to 1, then we have the desired result that $\Gamma^* < -S^* = -s^*/(1-\delta) \to -\infty$ as $\delta \to 1$. Thus, the remaining case is that $S^* + \Gamma^* \geq 0$ when $\delta \to 1$. Given this, if δ is close to 1, by using the fact that $\Phi_t = \delta S^* - \delta u$ is a feasible choice in Problem M, we can show that

$$\begin{aligned}
\Gamma^* &= c - \delta S^* + (1-p(1,1))\Phi^* \\
&\quad + q(\hat{a})\max\{\delta S^* + \delta\Gamma^*, 0\} + (1-q(\hat{a}))\max\{\delta S^* - \Phi^* + \delta\Gamma^*, 0\} - c(\hat{a}) \\
&\leq c - \delta S^* + (1-p(1,1))\{\delta S^* - \delta u\} \\
&\quad + q(\hat{a})\max\{\delta S^* + \delta\Gamma^*, 0\} + (1-q(\hat{a}))\max\{\delta u + \delta\Gamma^*, 0\} - c(\hat{a}) \\
&= c - \delta S^* + (1-p(1,1))\{\delta S^* - \delta u\} + q(\hat{a})\{\delta S^* + \delta\Gamma^*\} - c(\hat{a}),
\end{aligned}$$

which shows that

$$\Gamma^* \leq \frac{1}{1-\delta q(\hat{a})}\{c - \Delta q(\hat{a})\delta S^* - (1-p(1,1))\delta u - c(\hat{a})\}.$$

The RHS of the above inequality tends to $-\infty$ as $\delta \to 1$ for all $\hat{a} \in \{0,1\}$.

8.4 Proof of the sufficiency part in Proposition 2

We show that conditions (6) and (7) stated in Proposition 2 are sufficient for the first best to be attained under hierarchy. We do this by constructing equilibrium strategies under hierarchy that implement the first best. Suppose (6) and (7) are satisfied.

Let $b_t^1 = \hat{b}^1(y_t, y^{t-1})$ be the net informal bonus for agent 1 in period t. Consider now the stationary transfers for agent 1 such that $\hat{b}^1(Y, y^{t-1}) = 0$ and $\hat{b}^1(0, y^{t-1}) = -\Phi^*$. Thus $\hat{b}^1(y, y^{t-1}) \leq 0$ for all $y \in \{0, Y\}$ and all y^{t-1}. We suppress y^{t-1} from $\hat{b}^1$ in what follows. Next, set the discounted present value of agent 1's payoffs as $U^1(y^{t-1}) = U^1 \equiv S^*$ for all y^{t-1}. Given this, we specify the net up-front transfer for agent 1 as $w_t^1(y^{t-1}) = \hat{w}^1$ satisfying the following:

$$U^1 = S^* = \hat{w}^1 + \delta S^* - (1 - p(1,1))\Phi^* - c.$$

Then $\hat{w}^1 = (1-\delta)S^* + (1-p(1,1))\Phi^* + c > 0$. Note that $\hat{b}^1(Y) + \delta U^1 = \delta S^* \geq \delta u$ and $\hat{b}^1(0) + \delta U^1 = -\Phi^* + \delta S^* \geq \delta u$ by the definition of Φ^*. Note also $b^1(Y) + \delta U^1 - \{\hat{b}^1(0) + \delta U^1\} = \Phi^* \geq 2c/\Delta p$ so that agent 1's IC is satisfied.

For agent 2, consider stationary transfers as follows: $b_t^2 = \hat{b}^2(y)$ and $w_t^2 = \hat{w}^2$ for all t such that $\hat{b}^2(Y) = c/\Delta p$, $\hat{b}^2(0) = 0$, and $\hat{w}^2 = -u$.

Based on the above, consider the following transfers: (i) $\hat{b}_{1,t}^p(y_t) = -\hat{b}^1(y_t) \geq 0$ and $\hat{b}_{p,t}^1(y_t) = 0$, (ii) $\hat{w}_{1,t}^p = 0$ and $\hat{w}_{p,t}^1 = \hat{w}^1$ regardless of y^{t-1}, (iii) $\hat{b}_{1,t}^2(y_t) = \hat{b}^2(y_t)$ and $\hat{b}_{2,t}^1(y_t) = 0$, and (iv) $\hat{w}_{1,t}^2 = 0$ and $\hat{w}_{2,t}^1 = -\hat{w}^2 = u$ regardless of y^{t-1}. We show below that these are the transfers on the equilibrium path supported by the strategy profile described below. In the following, we use the 'hat' to indicate on-the-path transfers, and denote transfers off-the-path without the 'hat'.

Strategy profile (**):

Principal's Strategy:

- If the principal and agent 1 have made the transfers stipulated above up to period $t-1$, then the principal pays $\hat{w}_{p,t}^1 = \hat{w}^1$ to agent 1.
- If the principal and agent 1 have made the stipulated transfers until the realization of y_t, then the principal pays $\hat{b}_{p,t}^1(y_t)$ to agent 1.
- If either the principal or agent 1 has not followed the stipulated transfers in the past before period t, then the principal chooses $w_{p,t}^1 = 0$ and exercises the default option against agent 1 in period t.

Agent 1's Strategy:

- If the principal and both agents have made the stipulated transfers up to period $t-1$, then agent 1 chooses $\hat{w}_{1,t}^2 = 0$ and $\hat{w}_{1,t}^p = 0$ in period t.
- Following the above, if the principal does not exercise the default option in period t, then agent 1 chooses $a_{1,t} = 1$ in period t.
- If the principal and both agents have made the stipulated transfers until the realization of y_t, then agent 1 pays $\hat{b}_{1,t}^2(y_t)$ to agent 2 and $\hat{b}_{1,t}^p(y_t)$ to the principal.
- Suppose either agent 1 or agent 2 has not made the stipulated payments to each other in the past before period t. Then agent 1 continues to receive $\hat{w}^1 > 0$ from the principal, and chooses the optimal effort $\hat{a}$ which maximizes $w^1 + E_y[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a, 0] - c(a)$, and obtains the maximum value $\tilde{U}^1 = \Gamma^* + U^1 = \Gamma^* + S^*$,

while paying $\hat{w}^2_{1,t} = \hat{b}^2_{1,t} = 0$ to agent 2. In this case, since $\Gamma^* + S^* > 0$,[14] agent 1 continues to transact with the principal even after he has made a secret deviation against agent 2.

- Suppose either agent 1 or the principal has not made the stipulated transfers in the past before period t. Then agent 1 pays $w^p_{1,t} = 0$ to the principal and chooses $a_{1,t} = 0$ in period t. In addition, agent 1 pays $w^2_{1,t} = 0$ to agent 2 and exercises the default option against agent 2 in period t.

- Suppose the principal and both agents have made the stipulated transfers up to period $t-1$ and $w^1_t = \hat{w}^1$, but $w^2_t \neq \hat{w}^2$. Then agent 1 exercises the default option against agent 2 and chooses the optimal effort $\hat{a}$ that maximizes $E_y[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a, 0] - c(a)$, following the stipulated informal bonus $\hat{b}^1(y_t)$ in period t.

- Suppose the principal and both agents have made the stipulated transfers up to period $t-1$, but $w^1_t \neq \hat{w}^1$. Then, agent 1 chooses $a_{1,t} = 0$, exercises the default option against agent 2, and pays him $b^2_{1,t}(y_t) = 0$.

Agent 2's Strategy:

- If both agents have made the stipulated transfers up to period $t-1$, then agent 2 pays $\hat{w}^1_2 = u$ to agent 1; otherwise, he pays nothing to agent 1 and chooses $a_{2,t} = 0$.

- Suppose both agents have made the stipulated transfers up to period $t-1$ and $\hat{w}^2$. If agent 1 does not exercise the default option in period t, then agent 2 chooses $a_{2,t} = 1$; otherwise, he chooses $a_{2,t} = 0$.

- Regardless of the past history, agent 2 pays $b^1_{2,t}(y_t) = 0$ to agent 1 no matter what $y_t \in \{Y, 0\}$ is realized.

We now show that the above strategies and stipulated transfers constitute the equilibrium that implements the first best. We do this by checking incentive compatibility and dynamic enforcement constraints for the principal and the two agents.

First, it is easy to check that incentive compatibility is satisfied for both agents. For example, (IC$_1$) is satisfied because $\hat{b}^1(Y) + \delta U^1 - \{\hat{b}^1(0) + \delta U^1\} = \Phi^* \geq 2c/\Delta p = c/\Delta p + \hat{b}^2(Y) - \hat{b}^2(0)$.

Next, we check agent 1's dynamic enforcement constraint (DEA$_1$). If agent 1 deviates against agent 2 in period t by not paying $\hat{b}^2$ ($b^2_t \neq \hat{b}^2$), then he exercises the default option against agent 2 in period $t+1$ according to the prescribed strategy (**). Thus $a_{2,s} = 0$ for all $s \geq t+1$. Also, if agent 1 deviates further against the principal by not paying $\hat{b}^1$ ($b^1_t \neq \hat{b}^1$), then the prescribed strategy (**) indicates that the principal makes no transfer to agent 1 and exercises the default option against him from period $t+1$. Agent 1's best response to this is to make no transfer to the principal and choose low effort from

[14]Suppose that $\Gamma^* + S^* \leq 0$. Then we obtain $\Gamma^* = c - \delta S^* + (1 - p(1,1))\Phi^*$, which however implies that $\Gamma^* + S^* = c + (1-\delta)S^* + (1 - p(1,1))\Phi^* > 0$, a contradiction. Thus $\Gamma^* + S^* > 0$.

period $t+1$. This gives agent 1 payoff of $\hat{w}_p^1 + \delta \times 0 = \hat{w}^1 > 0$ in period t. Alternatively, agent 1 can continue to follow the stipulated transactions with the principal even after the deviation against agent 2 while optimally choosing his efforts over time. Thus, agent 1's deviation payoff is the larger of the payoffs from these two options. In sum, after the deviation against agent 2 in period t (that is, $b_t^2 \neq \hat{b}^2$), agent 1 obtains the following continuation value as the deviation payoff from period $t+1$:

$$\tilde{U}^1 = \max\left\{\hat{w}^1, \hat{w}^1 + \max_{a\in\{0,1\}} E[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a, 0] - c(a)\right\}.$$

This implies

$$\tilde{U}^1 = \hat{w}^1 + \max_{a\in\{0,1\}} \{E[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a, 0] - c(a)\}$$

because $\hat{w}^1 \geq 0$ and agent 1 can always choose $a = 0$, which implies that the maximum in the above expression over $a \in \{0, 1\}$ is non-negative.

Let $\phi^* \equiv \delta S^* - \Phi^*$. Then, by using the definition of $\hat{w}^1$ and $\hat{b}^1$ as well as $\Gamma^* \equiv \tilde{U}^1 - U^1$, we can re-write $\tilde{U}^1$ as follows:

$$\begin{aligned}
\tilde{U}^1 &= U^1 + c - p(1,1)\Phi^* - \phi^* \\
&\quad + \max_{a\in\{0,1\}} \{q(a)\max\{\hat{b}^1(Y) + \delta\tilde{U}^1, 0\} + (1-q(a))\max\{\hat{b}^1(0) + \delta\tilde{U}^1, 0\} - c(a)\} \\
&= U^1 + c - p(1,1)\Phi^* - \phi^* \\
&\quad + \max_a \{q(a)\max\{\Phi^* + \phi^* + \delta\Gamma^*, 0\} + (1-q(a))\max\{\phi^* + \delta\Gamma^*, 0\} - c(a)\} \\
&= U^1 + c - \delta S^* + (1-p(1,1))\Phi^* \\
&\quad + \max_a \{q(a)\max\{\delta S^* + \delta\Gamma^*, 0\} + (1-q(a))\max\{\delta S^* - \Phi^* + \delta\Gamma^*, 0\} - c(a)\}.
\end{aligned}$$

This leads to

$$\begin{aligned}
\tilde{U}^1 - U^1 &= \Gamma^* \\
&= c - \delta S^* + (1-p(1,1))\Phi^* \\
&\quad + \max_a q(a)\{\max\{\delta S^* + \delta\Gamma^*, 0\} \\
&\quad + (1-q(a))\max\{\delta S^* - \Phi^* + \delta\Gamma^*, 0\} - c(a)\}.
\end{aligned}$$

Then, $-\hat{b}^2(y) \geq \delta\Gamma^*$ implies that (DEA_1) is satisfied: $-b^2(y) + \delta U^1 \geq \delta\tilde{U}^1$ for any $y \in \{0, Y\}$.[15]

Next, it is straightforward to check that agent 1 cannot profitably deviate against both the principal and agent 2. If he does, then the prescribed strategy (**) implies that agent 1's continuation value following the deviation is zero. By not deviating, he obtains $-\hat{b}^2(Y) + \hat{b}^1(Y) + \delta U^1 \geq 0$ if $y_t = Y$, and $-\hat{b}^2(0) + \hat{b}^1(0) + \delta U^1 = \delta S^* - \Phi^* \geq 0$ if $y_t = 0$.

Consider now agent 1's deviation against the principal ($b^1(y_t) \neq \hat{b}^1(y_t)$), to which the principal responds according to the prescribed strategy (**). After the deviation, agent 1 has two options in contracting with agent 2. First, he pays $\hat{w}^2$ to agent 2 but chooses

[15]Since $\Gamma^* < 0$, (DEA_1) is slack when $y = 0$: $-\hat{b}^2(0) = 0 > \delta\Gamma^*$.

$a_{1,t+1} = 0$, followed by paying $b^2_{1,t+1} = 0$ to agent 2, which gives him $-\hat{w}^2 = u$ in period $t+1$ and the payoff of zero in all subsequent periods $s \geq t+2$. Second, he continues to make the stipulated transfer to agent 2 from period $t+1$ onward, which gives him at most

$$\max_{a \in \{0,1\}} -\{\hat{w}^2 + E[\hat{b}^2(y)|a,1]\} - c(a)$$

each period after period $t+1$. The payoff from the second option is not larger than $-c < 0$. Thus agent 1 chooses the first option after the deviation against the principle, which gives him the continuation value $\delta(-\hat{w}^2) = \delta u$. However, since $\hat{b}^1(Y) + \delta U^1(Y) = \delta S^* \geq \delta u$ and $\hat{b}^1(0) + \delta U^1(0) = \delta S^* - \Phi^* \geq \delta u$, it is not profitable for agent 1 to deviate from $\hat{b}^1(y_t)$ while maintaining $\hat{b}^2(y_t)$ in period t.

Next, we check agent 2's deviation incentives. First, it is easy to verify that his dynamic enforcement constraint (DEA_2) is satisfied: $\hat{b}^2(y_t) + \delta U^2 = \hat{b}^2(y_t) \geq 0$. Thus, agent 2 has no incentives to deviate from the stipulated bonus. Second, if agent 2 deviates from $\hat{w}^2 = -u$, then agent 1 will immediately exercise the default option against agent 2. This gives agent 2 at most the payoff of zero. If agent 2 follows the stipulated up-front transfer $\hat{w}^2 = -u$, then he obtains the equilibrium value of $U^2 = 0$.

We now turn to the principal's dynamic enforcement constraint (DEP). Let V denote the discounted present value of the principal's payoff when all players follow the stipulated payments in every period. Then, we need $\delta V \geq \hat{b}^1(Y)$. This is equivalent to $\delta(S^* - U^1) \geq \hat{b}^1(Y)$, which can be written as $\delta S^* \geq \hat{b}^1(Y) + \delta U^1$. This is satisfied as equality due to the definition of $\hat{b}^1$ and U^1. Also, $\delta V \geq \hat{b}^1(0)$ because $\delta S^* \geq \hat{b}^1(0) + \delta U^1 = \delta S^* - \Phi^*$ and $V = S^* - U^1$.

Finally, we verify the optimal choice of up-front transfers for each player. First, consider agent 1. Following the prescribed strategy (**), agent 1 receives positive net up-front transfers from both the principal and agent 2: $\hat{w}^1 > 0, \hat{w}^2 = -u < 0$. Agent 1 does not have incentives to deviate from $\hat{w}^p_1 = 0$ because doing so results in his continuation value of zero, as shown above, but the expected continuation bonus is non-negative, i.e., $E[\hat{b}^1(y) + \delta U^1|1,1] \geq c > 0$, due to ($\text{IC}_1$).[16] Likewise, agent 1 does not have incentives to deviate from $\hat{w}^2_1 = 0$. His continuation value after deviation is $E[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a,0] - c(a)$. As we have seen above, $\Gamma^* < 0$ so that $U^1 \geq \tilde{U}^1$, and hence

$$\begin{aligned} U^1 &= \hat{w}^1 + E[\hat{b}^1(y) + \delta U^1|1,1] - c \\ &> \tilde{U}^1 \\ &> \hat{w}^1 - w^2_1 + \max_{a \in \{0,1\}} \{E[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a,0] - c(a)\} \end{aligned}$$

for any deviation $w^2_1 > \hat{w}^2_1 = 0$. This shows that $U^1 > \hat{w}^1 - w^2_1 + \max_a E[\max\{\hat{b}^1(y) + \delta\tilde{U}^1, 0\}|a,0] - c(a)$. Thus agent 1 cannot make any profitable deviation. Second, consider the principal. If the principal deviates from paying $\hat{w}^1_p = \hat{w}^1 > 0$, then by the prescribed

[16] Note that IC_1 implies that $E[\hat{b}^1(y)|1,1] + \delta U^1 - c \geq E[\hat{b}^1(y)|1,0] + \delta U^1 = \delta S^* - (1 - p(1,0))\Phi^* \geq \delta S^* - \Phi^* \geq 0$.

strategy (**), the principal's continuation value is zero. Since the equilibrium value of the principal under the prescribed strategy (**) is given by $V = S^* - U^1 = 0$ for each $y \in \{0, Y\}$, the above deviation cannot be profitable. Third, if agent 2 deviates against agent 1 by paying any $w^1_{2,t} \geq 0$, then he obtains $\hat{w}^2_1 - w^1_{2,t} = -w^1_{2,t} < 0$ based on the prescribed strategy (**). By following the stipulated transfer, however, agent 2 obtains $U^2 = 0$. Thus, none of the players has incentives to deviate from the prescribed up-front transfers. Q.E.D.

8.5 Proof of the remark at the end of Section 4

Let $\{\{\hat{b}^i_t\}_i, \{\hat{w}^i_t\}_i\}^\infty_{t=1}$ be equilibrium net transfers that support the first best outcome under centralization or hierarchy.

First, consider centralization. Suppose that $(a_1, a_2) = (1, 0)$ is implemented every period so that $s(1, 0)/(1 - \delta)$ is attained in period 1. Let $\{V_t, \{U^i_t\}_i\}^\infty_{t=1}$ be the sequence of equilibrium values for the principal and agents. Then it must be that $\delta V_2(Y) \geq \hat{b}^1(Y)$ when $y_1 = Y$ is realized in period 1. This implies that $\delta\{V_2(Y) + \sum_i U^i_2(Y)\} \geq \hat{b}^1(Y) + \delta U^1_2(Y) + \delta U^2_2(Y)$. In the above, the LHS must be equal to $S(1, 0) \equiv s(1, 0)/(1 - \delta)$ whereas the RHS satisfies

$$\begin{aligned}
\hat{b}^1_1(Y) + \delta U^1_2(Y) + \delta U^2_2(Y) &\geq \hat{b}^1_1(0) + \delta U^1_2(0)c/\Delta p + \delta U^2_2(Y) \\
&\geq c/\Delta p
\end{aligned}$$

due to (IC_1) in period 1, that is, $b^1_1(Y) + \delta U^1_2(Y) \geq c/\Delta p + b^1_1(0) + \delta U^1_2(0)$, and the fact that $U^2_2(Y) \geq 0$. Thus $\delta S(1, 0) \geq c/\Delta p$ must be satisfied. Conversely, if this holds, then we can construct the strategies that support $S(1, 0)$ as an equilibrium as follows: $b^1(Y) = c/\Delta p$, $b^1(0) = 0$ and $w^1 = -u$ while $b^2(Y) = b^2(0) = w^2 = 0$.

Next, consider hierarchy. Suppose $a_1 = 1$ and $a_2 = 0$ for any $t \geq 1$. Then it must be that $\delta V_2(Y) \geq \hat{b}^1(Y)$ in period 1. Then we have

$$\begin{aligned}
\delta S(1, 0) &= \delta \left\{ V_2(Y) + \sum_i U^i_2(Y) \right\} \\
&\geq \hat{b}^1_1(Y) + \sum_i U^i_2(Y) \\
&\geq c/\Delta p + \{\hat{b}^1_1(0) - \hat{b}^2_1(0) + \delta U^1_2(0)\} + \{\hat{b}^2_1(Y) + \delta U^2_2(Y)\} \\
&\geq c/\Delta p
\end{aligned}$$

due to (IC_1) in period 1, (DEA_1) corresponding to $y_1 = 0$, and (DEA_2) corresponding to $y_1 = Y$. Thus $\delta S(1, 0) \geq c/\Delta p$. Suppose next that $a_1 = 0$ and $a_2 = 1$ for any $t \geq 1$. (DEA_1) must be satisfied in period 1 as follows: $\delta U^1_2(Y) \geq b^2_1(Y)$. Then, by using ($\mathrm{IC}_1$) and ($\mathrm{DEA}_1$) in period 1, we have

$$\begin{aligned}
\delta \sum_i U^i_2(Y) &\geq \hat{b}^2_1(Y) + \delta U^1_2(Y) \\
&\geq c/\Delta p + \hat{b}^2_1(0) + \delta U^1_2(0) \\
&\geq c/\Delta p.
\end{aligned}$$

Since $V_2(Y) \geq 0$, we have in period 1

$$\begin{aligned} \delta S(1,0) &= \delta \left\{ V_2(Y) + \sum_i U_2^i(Y) \right\} \\ &\geq \delta \sum_i U_2^i(Y) \\ &\geq c/\Delta p, \end{aligned}$$

which yields $\delta S(1,0) \geq c/\Delta p$.

Conversely, if $\delta S(1,0) \geq c/\Delta p$, then we can consider the hierarchy in which agent 1 chooses $a_1 = 1$ and agent 2 chooses $a_2 = 0$. In this case, the dynamic game is reduced to the bilateral contracting relationship in which only the principal and agent 1 interact with each other over time, whence we can construct the strategies of the principal and agent 1 that support $\delta S(1,0)$.

9 Remarks on the Case with Observable Decisions

In the main text, we assumed that output is the only publicly observable variable, and the decisions on transfers and default options are observed only by the two parties directly involved in the relevant bilateral relationship. In this section, we consider the case where these decisions are also publicly observed so that effort choice by the agents is the only private information. In this case, we show that centralization and hierarchy are equivalent in terms of the critical discount factors necessary to achieve the first best.

We start by providing some intuition. Consider centralization first. The assumed public information allows multilateral punishment when the principal deviates against one agent. Thus, the first best is attained under centralization if and only if the discounted future value of the first-best surplus is not less than than total incentive costs for both agents: $\delta S^* \geq 2c/\Delta p$. A similar reasoning applies to hierarchy. The principal's deviation against agent 1 triggers punishment by agent 1, which in turn triggers punishment by agent 2. Likewise, agent 1's deviation against agent 2 induces the principal to punish agent 1, leading all players to move to the punishment phase. Based on this intuition, we can show the following result.

Proposition S1. *Suppose that all up-front transfers, informal bonuses and decisions on the default option are observable to all players. Then the first best is sustained under centralization or hierarchy if and only if*

$$\delta S^* \equiv \frac{\delta s^*}{1-\delta} \geq 2c/\Delta p$$

Proof: First, consider centralization. Suppose that there exists an equilibrium which attains S^* in period 1, and let $b_1^i(Y)$ be the equilibrium net transfer for agent i in period 1 when $y_1 = Y$. Let $V_2(y_1)$ and $U_2^i(y_1)$ be the equilibrium payoffs for the principal and agent i in period 2, conditional on $y_1 \in \{Y, 0\}$. Since the principal should not deviate from $b_1^i(Y)$ against either agent, we must have

$$\delta V_2(Y) \geq \sum_i b_1^i(Y).$$

By adding $\sum_i \delta U_2^i(Y)$ to both sides of the above inequality, we obtain the dynamic enforcement constraint for the principal (DEP):

$$\delta \left\{ V_2(Y) + \sum_i U_2^i(Y) \right\} \geq \sum_i \{b_1^i(Y) + \delta U_2^i(Y)\}.$$

Next, we can write agent i's incentive compatibility and dynamic enforcement constraints in period 1 as follows: (IC$_i$) $b_1^i(Y) + \delta U_2^i(Y) \geq c/\Delta p + b_1^i(0) + \delta U_2^i(0)$; (DEA$_i$) $b_1^i(0) + \delta U_2^i(0) \geq 0$ when $y_1 = 0$. Since $S^* = V_2(Y) + \sum_i U_2^i(Y)$ in the equilibrium sustaining the first best, the above (DEP) can be expressed as

$$\begin{aligned} \delta S^* &\geq 2c/\Delta p + \sum_i \{b_1^i(0) + \delta U_2^i(0)\} \\ &\geq 2c/\Delta p. \end{aligned}$$

We now verify that the above condition is also sufficient for the first best to be attained under centralization. Set transfers as follows: (i) the principal pays $b_{p,t}^i(Y) = c/\Delta p$ and $b_{p,t}^i(0) = 0$, and $w_{p,t}^i = 0$ for any $t \geq 1$; (ii) agent i pays $b_{i,t}^p(Y) = b_{i,t}^p(0) = 0$ and $w_{i,t}^p = u$. Then the net transfers are $b^i(Y) = c/\Delta p$, $b^i(0) = 0$, and $w^i = -u$ for $i = 1, 2$. Given these transfers, consider the following strategies: (i) players make the specified transfers as long as no player has deviated from these transfers in the past; (ii) if any player has deviated from any part of the above transfers in the past, the principal pays nothing to both agents and exercises the default option against both agents, and agents choose low effort. It is easy to check that the above strategies support the first best surplus S^* in period 1 if $\delta S^* \geq 2c/\Delta p$.

Next, consider hierarchy. Suppose that there exists an equilibrium which attains S^* in period 1, and let $b_1^i(Y)$ be the equilibrium net transfer for agent i in period 1 when $y_1 = Y$. Let $V_2(y_1)$ and $U_2^i(y_1)$ be the equilibrium payoffs for the principal and agent i in period 2, conditional on $y_1 \in \{Y, 0\}$. Since the principal should not not deviate from $b_1^1(Y)$, we have the following (DEP):

$$\delta V_2(Y) \geq b_1^1(Y).$$

Additionally, (IC$_1$) in period 1 is given by $b_1^1(Y) + \delta U_2^1(Y) - b_1^2(Y) \geq c/\Delta p + b_1^1(0) + \delta U_2^1(0) - b_1^2(0)$, and (DEA$_1$) in period 1 is $-b_1^2(0) + b_1^1(0) + \delta U_2^1(0) \geq 0$ when $y_1 = 0$. Similarly, (IC$_2$) in period 1 is $b_1^2(Y) + \delta U_2^2(Y) \geq c/\Delta p + b_1^2(0) + \delta U_2^2(0)$, and (DEA$_2$) in period 1 is $b_1^2(0) + \delta U_2^2(0) \geq 0$ when $y_1 = 0$. By adding $\sum_i U_2^i(Y)$ to both sides of (DEP), we have

$$\begin{aligned}
\delta S^* &= \delta\left\{V_2(Y) + \sum_i U_2^i(Y)\right\} \\
&\geq b_1^1(Y) + \delta U_2^1(Y) + \delta U_2^2(Y) \\
&\geq c/\Delta p + b_1^2(Y) - b_1^2(0) + b_1^1(0) + \delta U_2^1(0) + \delta U_2^2(Y) \\
&\geq c/\Delta p + b_1^2(Y) + \delta U_2^2(Y) \\
&\geq 2c/\Delta p + b_1^2(0) + \delta U_2^2(0) \\
&\geq 2c/\Delta p,
\end{aligned}$$

which gives us the necessary condition for S^* to be attained under hierarchy.

We show next that the above necessary condition is also sufficient. To this end, consider the following *semi-stationary* contract: (i) $\hat{b}^1(Y) = c/\Delta p$ and $\hat{b}^1(0) = 0$, and $w_t^1 = \hat{w}^1(Y) \equiv c/\Delta p\delta - 2p(1,1)(c/\Delta p) + c$ if $y_{t-1} = Y$ and $w_t^1 = \hat{w}^1(0) \equiv -2p(1,1)c/\Delta p + c$ if $y_{t-1} = 0$; (ii) $\hat{b}^2(Y) = c/\Delta p$ and $\hat{b}^2(0) = 0$, and $\hat{w}^2 = -u$. Note that $w^1(y_{t-1})$ depends on output y_{t-1}. Given these transfers, the equilibrium values of agent 1's payoffs are

$$U^1(Y) = w^1(Y) + p(1,1)\hat{b}^1(Y) - c + p(1,1)\delta U^1(Y) + (1 - p(1,1))\delta U^1(0)$$

and

$$U^1(0) = w^1(0) + p(1,1)\hat{b}^1(Y) - c + p(1,1)\delta U^1(Y) + (1 - p(1,1))\delta U^1(0).$$

Thus, we have $U^1(Y) = c/\Delta p\delta$ and $U^1(0) = 0$. Set $\hat{b}_{1,t}^p = \hat{b}_{2,t}^1 = 0$, $\hat{w}_{1,t}^2 = 0$ and $\hat{w}_{2,t}^1 = u$. Also set $\hat{w}_{1,t}^p = -\hat{w}^1(y_{t-1})$ and $\hat{w}_{p,t}^1 = 0$ if $w^1(y_{t-1}) < 0$, and $\hat{w}_{1,t}^p = 0$ and $\hat{w}_{p,t}^1 = \hat{w}^1(y_{t-1})$ if $\hat{w}^1(y_{t-1}) \geq 0$.

Given the above transfers, consider the following strategies. When all players have followed the prescribed payments without exercising the default option in all the past periods, they make the stipulated transfers without exercising the default option in the current period. In addition to this on-the-path strategies, we specify off-the-path strategies as follows.

Principal's Strategy:

- If any player has deviated from the stipulated transfers before period t, then the principal pays $w_{p,t}^1 = 0$ to agent 1 and exercises the default option.
- If any player has deviated from the stipulated transfers until the realization of y_t, then the principal pays $b_{p,t}^1 = 0$ to agent 1.

Agent 1's Strategy:

- If any player has deviated from the stipulated transfers before period t, then agent 1 pays $w_{1,t}^p = w_{1,t}^2 = 0$. In addition, if any player did not follow the stipulated up-front transfers or exercised the default option in period t, then agent 1 chooses $a_{1,t} = 0$.

- If any player has deviated from the stipulated transfers until the realization of y_t, then agent 1 pays $b^p_{1,t} = b^2_{1,t} = 0$.

Agent 2' Strategy:

- If any player has deviated from the stipulated transfers before period t, then agent 2 pays $w^1_{2,t} = 0$. In addition, if any player exercised the default option or deviated from the stipulated up-front transfers in period t, then agent 2 chooses $a_{2,t} = 0$.
- Agent 2 pays the stipulated bonus $\hat{b}^1_{2,t}(y_t) = 0$ to agent 1 regardless of the past history.

We now show that the above strategies constitute an equilibrium. First, if the principal deviates from $\hat{b}^1(Y)$ when $y_t = Y$, then both agents will choose low effort from the next period onwards, which gives her continuation value of zero. Since the principal's equilibrium value is $V(Y) = S^* - U^1(Y) = S^* - U^1(Y) = S^* - c/\Delta p\delta$, we obtain $\delta V(Y) = \delta S^* - c/\Delta p \geq \hat{b}^1(Y) = c/\Delta p$. Thus, the principal cannot make profitable deviations when $y_t = Y$. Second, if agent 1 deviates from $\hat{b}^2(Y)$, then his continuation value is zero from the next period given the above strategies. But we have $-\hat{b}^2(Y) + \delta U^1(Y) = -c/\Delta p + c/\Delta p = 0$, hence agent 1 cannot make profitable deviations. Third, we obtain (IC_1) as follows: $\hat{b}^1(Y) + \delta U^1(Y) - \hat{b}^2(Y) \geq \hat{b}^1(0) + \delta U^1(0) - \hat{b}^2(0) + c/\Delta p$. Finally, if the principal or agent 1 does not follow $\hat{w}^1(y_{t-1})$, then the principal will exercise the default option against agent 1, and both agents choose low effort forever. Thus neither the principal nor agent 1 can benefit from such deviations. If agent 1 or agent 2 deviates from $\hat{w}^2$, then the principal will exercise the default option against agent 1 and both agents will choose low effort forever. Thus such deviations are not profitable for either agent. Q.E.D.