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Discussion Paper no. [2023-13](#)**Siew Ling Yew and Jie Zhang****Abstract:**

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Keywords: Health externality, Longevity, Productivity, Fertility, Savings**JEL Classification:** H21, I13, I15

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Health externalities to productivity and efficient health subsidies[♣]

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1. Introduction

Health is an important determinant of wellbeing and productivity with internal and external benefits (e.g., Strauss and Thomas, 1998; Bloom and Canning, 2000). Empirical evidence indicates a positive effect of health spending on health outcomes (Wolfe and Gabay, 1987; Hitiris and Posnett, 1992; Cremieux et al., 1999; Or, 2000; Kim and Lane, 2013). Empirical evidence also suggests substantial economic costs of poor health and external benefits of health improvements through preventive and curative health care (Loeb et al., 2010; Bloom et al., 2020a; White, 2021; de Courville et al., 2022). For example, infectious diseases lead to adverse macroeconomic effects (Bloom et al., 2020a) and workplace productivity losses (due to presenteeism or absenteeism with or without sick leave) as well as mortality unrecognized by individuals (de Courville et al., 2022). Influenza vaccination provides substantial external benefits by reducing influenza-related mortality and illness-related work absences (White, 2021; Acton et al., 2022). However, market failure occurs when firms benefit from the productivity of healthy workers but cannot directly observe individuals' health status (Sauermann, 2016). Without a link of the wage to a worker's health status, workers overlook the contributions of their health investment to average productivity.

When workers' health status has an external contribution to average productivity,¹ the social return to health spending is higher than the private return. Thus, health externalities to productivity lead to low health spending which may in turn cause low productivity (the opportunity cost of time raising a child), longevity, savings and labor but high fertility as in developing countries. By contrast, most OECD countries have universal public healthcare and US employers often subsidize health insurance for workers. The public or firms' health subsidies may lead to high longevity, savings, productivity and low fertility.

However, health externalities to productivity are often absent in models of health, savings, fertility. For instance, better health raises labor, income, longevity, and savings but lowers fertility (Bloom et al., 2007; Bloom et al., 2020b). Public health spending increases longevity, savings, growth, and welfare (Chakraborty, 2004). Public health subsidies enhance longevity and welfare but reduce savings and future output without old-age labor (Zhang et al., 2006). Optimal health subsidies depend negatively on public pensions (Pestieau et al., 2008). In an R&D-based growth model (Kuhn and Pretzner, 2016), expanding public healthcare beyond the growth-maximizing level can be Pareto superior. With a negative externality of longevity on

¹ This externality is in line with empirical evidence that workers' productivity rises when coworkers are more productive in the workplace (e.g., Falk and Ichino, 2006).

annuity returns and without old-age labor, combining social security with public healthcare can attain socially optimal outcome (Yew and Zhang, 2018). Studies on employer-based health insurance against individual health shocks explore its effects on wage, labor, and welfare (e.g., Dey and Flinn, 2005; Jeske and Kitao, 2009; Feng and Zhao, 2018; Feng and Villamil, 2022).

This study aims to investigate health spending, fertility, labor supply, savings, longevity and appropriate taxes and public or firms' health subsidies to attain the socially optimal allocation in a lifecycle-dynastic model with health externalities to productivity. Departing from lifecycle models where health spending only extends longevity and retirement lives, health spending here has external benefits for the average health and productivity of the labor force and private benefits for longevity and working life at old age. The dynastic model also has two-sided intergenerational transfers in contrast to standard overlapping-generations models.² The costs of raising a child consist of forgone earnings for time-intensive childrearing, bequests, and inter-vivos transfers between young and old parents.³

This study makes the following contributions. First, health externalities to productivity cause low health spending, longevity, productivity, labor and savings but high fertility because perceived private returns to health spending are below the social return. Second, appropriate taxes and subsidies can attain the socially optimal outcome through age-specific rates of health subsidies financed by age-specific tax rates on labor income. Intuitively, health subsidies increase health spending by lowering the private cost of health spending. The rise in health spending, particularly at young age, increases longevity, old-age labor and average productivity (the time cost of rearing a child), thus lowering fertility and raising young-age labor. However, taxes on young-age labor income reduce the time cost of childrearing (after-tax earnings), thus increasing fertility and decreasing young-age labor. Tax deductions or subsidies on old-age labor income encourage old-age labor and health spending, thus increasing parental transfers to children (transfer cost of raising a child) or decreasing transfers from young to old parents.⁴

It is also optimal to finance health subsidies by consumption taxes and opposite taxes or subsidies on savings and capital income. Intuitively, consumption taxes reduce the relative cost of childrearing. Subsidies on savings raise the return to savings and reduce the bequest cost of raising a child, while taxes on capital income reduce the after-tax return to savings.

² Existing evidence finds intergenerational transfers within families (Laitner and Juster, 1996; Altonji et al., 1997).

³ For instance, altruistic parents are willing to ease the tax burdens of public transfers to the elderly on children by giving bequests as in Zhang (1995).

⁴ Weinzierl (2011) argues that age-dependent taxes are useful for Pareto improvements in a model with private information for earnings ability in the absence of investment in capital and health.

Third, firms' health subsidies in terms of employer-based health insurance for profit maximization mitigate the efficiency loss of the health externalities by increasing health spending, thus increasing longevity and productivity (the time cost of raising a child). As the positive effect of firms' health subsidies on the cost of raising a child decreases fertility, firms' health subsidies alone cannot attain socially optimal fertility. Thus, firms' health subsidies and appropriate taxes can achieve the socially optimal outcome when their opposite effects on the cost of raising a child just correct the excessive fertility that arises from the health externality. Firms' health subsidies also ease the financial burden of universal public healthcare.

For quantitative implications, we calibrate the model to match observed longevity, savings, fertility, output per capita, health-spending shares of output, taxes, subsidies and public transfers to the elderly in Australia with available data and a universal healthcare system. From the Australian tax system, the levels of young- and old-age health spending, young-age consumption, longevity, capital per worker and output per worker are above their *laissez-faire* levels but below their socially optimal levels. However, old-age consumption is above its socially optimal level, while fertility is lower but young-age labor is higher than their *laissez-faire* and socially optimal levels. The low fertility is possibly due to low savings subsidies and high public transfers to the elderly in Australia. The results suggest policy improvements in Australia such as raising health and savings subsidies and consumption taxes or reducing labor-income taxes and public transfers to the elderly.

The results in this study are broadly consistent with available empirical evidence. First, there is a positive association between health improvements and income or economic growth with a causal link running from income or growth to health (Ettner, 1996; Smith, 1999) or from health to income or growth (Bhargava et al., 2001; Weil, 2007). Second, there is a positive link between income, health, and longevity (van Doorslaer et al., 1997; Widman, 2003). Third, health relates positively with productivity (Strauss and Thomas, 1998; Bloom and Canning, 2000; Schultz, 2002; Bloom et al., 2022) and with labor supply, especially at old age (Au et al., 2005; Disney et al., 2006; Cai, 2010).

The rest of this study proceeds as follows. Section 2 introduces the model. Section 3 establishes the socially optimal allocation. Section 4 characterizes equilibrium allocations and derives socially optimal subsidies and taxes. Section 5 explores quantitative implications at steady states. Section 6 characterizes equilibrium allocations with firms' health subsidies and derives socially optimal taxes and firms' health subsidies. Section 7 presents discussions and conclusions. The Appendix contains all proofs.

2. The model

The model has an infinite number of periods. Each period has three overlapping generations: children, young and old parents. Children make no decision. The length of young parenthood equals one and that of old parenthood, $T(h_{t-1}, m_t): \mathbb{R}_+^2 \rightarrow (0,1)$, increases with health spending at young age h_{t-1} and at old age m_t at diminishing rates.

2.1. Households

The wellbeing of a dynasty increases with the consumption of the young parent c_t and the elderly $d_t T(h_{t-1}, m_t)$ and the number of children n_t at diminishing rates:

$$(1) \quad \sum_{t=0}^{\infty} \alpha^t \{ \beta U(T(h_{t-1}, m_t) d_t) + \alpha [U(c_t) + \rho G(n_t)] \},$$

with a subjective discount factor $\alpha \in (0,1)$ and tastes for old-age consumption $\beta \in (0,1)$ and the number of children $\rho > 0$. Raising a child needs $0 < v < 1$ fixed units of time with an upper bound on fertility, $n_t \leq 1/v$. Given wage rate w_t , each young parent allocates one unit of time endowment to rearing children vn_t and working $1 - vn_t$, receives a bequest from the old parent for $b_t > 0$, or gives a gift to the old parent for $b_t < 0$. The young parent allocates resources to young-age consumption c_t , health h_t , and savings s_t as follows:

$$(2) \quad c_t = b_t + (1 - vn_t)w_t - s_t - h_t.$$

Given gross interest rate (rental price of capital) R_t , an old parent spends wage income $T(h_{t-1}, m_t)w_t$ and capital income $R_t s_{t-1}$ on consumption and health, adjusted for old-age longevity, $(d_t + m_t)T(h_{t-1}, m_t)$, respectively, and leaves a bequest to or receives a gift from each child. Thus, the budget constraint of an old parent is

$$(3) \quad d_t T(h_{t-1}, m_t) = T(h_{t-1}, m_t)w_t + R_t s_{t-1} - m_t T(h_{t-1}, m_t) - b_t n_{t-1}.$$

This model abstracts from retirement at old age for simplicity. One can scale down old-age labor from longevity for retirement without changing the essence of results.

2.2. Production

The production function for a final good per young worker increases with capital per young worker k_t , labor per young worker l_t ,⁵ and the average health of the labor force $\bar{\Omega}(\bar{h}_t, \bar{m}_t)$:

$$(4) \quad y_t = f(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t),$$

with constant returns to scale in (k_t, l_t) . As noted in empirical evidence, the average health status of the labor force $\bar{\Omega}(\bar{h}_t, \bar{m}_t)$ depends positively on the average health spending by the young and old workers, \bar{h}_t and \bar{m}_t , respectively. One reason for the health externality to

⁵ Labor per young worker is the total labor of young and old workers over the number of young workers.

productivity is that firms do not perfectly observe the health status of individual workers, despite the contribution of their health to average productivity. Thus, firms do not directly link the wage for a worker to the worker's health status or health spending. Consequently, workers overlook the contributions of their health to average productivity, and thus their perceived private returns to health spending are lower than the social return. As one period here corresponds to 30 years, we assume that physical capital depreciates fully within one period.

Competitive firms compensate production factors by their marginal products as follows:

$$(5) \quad w_t = f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t),$$

$$(6) \quad R_t = f_k(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t).$$

The marginal products increase with the average health status of workers.

Markets clear when

$$k_{t+1} = s_t/n_t,$$

$$l_t = 1 - vn_t + T(h_{t-1}, m_t)/n_{t-1}.$$

The labor supply of all young workers $(1 - vn_t)n_{t-1}$ in the dynasty at time t decreases with fertility n_t but increases with the number of young workers n_{t-1} . The labor supply of the old worker $T(h_{t-1}, m_t)$ increases with lifetime health investment. The size of the young generation in the economy N_t evolves via $N_{t+1} = N_t n_t$. Feasibility for the allocation of output is

$$(7) \quad c_t = f(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) - k_{t+1}n_t - h_t - \frac{(d_t + m_t)T(h_{t-1}, m_t)}{n_{t-1}}.$$

To gauge the efficiency loss of the health externalities and to design optimal public policies to internalize the externalities, the next section explores the socially optimal allocation.

3. The socially optimal allocation

Given initial state (h_{-1}, k_0, n_{-1}) , the social planner chooses $\{d_t, m_t, c_t, n_t, k_{t+1}, h_t\}_{t=0}^{\infty}$ to maximize utility in (1) subject to feasibility in (7) by internalizing health externalities $h_t = \bar{h}_t$ and $m_t = \bar{m}_t$ as follows:

$$\begin{aligned} V(k_t, h_{t-1}, n_{t-1}) = & \max_{\{d_t, m_t, n_t, k_{t+1}, h_t\}} \{\beta U(T(h_{t-1}, m_t)d_t) + \\ & \alpha[U(f(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), 1 - vn_t + T(h_{t-1}, m_t)/n_{t-1}) - k_{t+1}n_t - h_t - \\ & (d_t + m_t)T(h_{t-1}, m_t)/n_{t-1}) + \rho G(n_t)] + \alpha V(k_{t+1}, h_t, n_t)\}. \end{aligned}$$

The planner chooses inter-vivos transfers to equate the marginal rate of substitution between the old and young agents' consumption with old-age dependency as follows:

$$(8) \quad \frac{\beta U'(T(h_{t-1}, m_t)d_t)}{\alpha U'(c_t)} = \frac{1}{n_{t-1}}.$$

High health spending in the previous period h_{t-1} contributes to old-age labor and longevity, thus motivating transfers from old to young agents that thin old-aged agents' consumption over old-age time and raise young agents' consumption. High fertility in the previous period means low old-age dependency, thus motivating net transfers from young to old agents.

The planner chooses investment in capital or bequests to children to equate the marginal rate of substitution between young-age consumption across generations with the marginal product of capital per young worker in the next period:

$$(9) \quad \frac{U'(c_t)}{\alpha U'(c_{t+1})} = \frac{f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})}{n_t}.$$

The average health of young and old agents in the next period contributes to the marginal product of capital, thus creating a positive substitution effect on savings or investment in capital.

When equalizing the marginal benefit and cost of young-age health spending, the planner internalizes young-age health externalities to productivity as follows:

$$(10) \quad \frac{T_h(h_t, m_{t+1}) \left[\beta U'(T(h_t, m_{t+1})d_{t+1})d_{t+1} + \frac{\alpha U'(c_{t+1})(f_l(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1}) - d_{t+1} - m_{t+1})}{n_t} \right]}{U'(c_t)[1 - f_{\bar{h}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)]} = 1.$$

The marginal cost of young-age health spending is the forgone marginal utility of young-age consumption. The marginal benefit is twofold in the next period: the marginal utility of old-age consumption from extended life and the marginal utility of children's young-age consumption from the increase in bequests to children owing to the extended working life at old age. The marginal product of average young-age health spending $f_{\bar{h}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)$ contributes to the marginal benefit of young-age health spending.

When equalizing the marginal gain and loss of old-age health spending, the planner internalizes old-age health externalities to productivity as follows:

$$(11) \quad \frac{T_m(h_{t-1}, m_t) \left[\beta U'(T(h_{t-1}, m_t)d_t)d_t + \frac{\alpha U'(c_t)(f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) - d_t - m_t)}{n_{t-1}} \right]}{\beta U'(T(h_{t-1}, m_t)d_t)[T(h_{t-1}, m_t) - n_{t-1}f_{\bar{m}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)]} = 1.$$

The marginal cost of old-age health spending is the forgone marginal utility of old-age consumption. The marginal benefit includes the marginal utility of old-age consumption from extended life and the marginal utility of children's young-age consumption from the increase in inter-vivos transfers from parents to children owing to the extended working life at old age. The marginal product of average old-age health spending $f_{\bar{m}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)$ contributes to the marginal benefit of old-age health spending.

The planner also equalizes the marginal rate of substitution between fertility and young-age consumption and their relative costs as follows:

$$(12) \quad \frac{\rho G'(n_t)}{U'(c_t)} = v f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) + k_{t+1} + \frac{\alpha U'(c_{t+1}) T(h_t, m_{t+1}) (f_l(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1}) - d_{t+1} - m_{t+1})}{U'(c_t) n_t^2}.$$

The first term in the relative cost of fertility is the time cost of childrearing; the second term is capital per child from parental investment (bequests); and the last term is the discounted marginal cost of inter-vivos transfers from parents to children in the next period. Particularly, old-age labor relates positively with lifetime health investment and interacts with capital accumulation, inter-vivos transfers and fertility in contrast to standard overlapping-generations models that assume retirement at old age without intergenerational transfers and old-age labor.

The transversality condition is

$$\lim_{t \rightarrow \infty} \alpha^t U'(c_t) n_t k_{t+1} = 0.$$

Denoting $f_k(t) \equiv f_k(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)$ and combining this transversality condition with successive substitutions on $\alpha U'(c_t) = u'(c_{t-1}) n_{t-1} / f_k(t)$ from condition (9) yields

$$\lim_{t \rightarrow \infty} k_{t+1} \prod_{j=0}^t \frac{n_j}{f_k(j)} = 0.$$

From this condition, the marginal products of capital should exceed the growth rates of aggregate capital for dynamic efficiency and a bounded value function of the state $V(\cdot, \cdot)$ when time approaches infinity.

The socially optimal allocation from an initial state (h_{-1}, n_{-1}, k_0) is a sequence $\{c_t, d_t, h_t, m_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ that satisfies technology (4), feasibility (7), first-order conditions (8) to (12) and the transversality condition. We now turn to the equilibrium allocations.

4. Equilibrium allocations

This section first determines the equilibrium allocation without firms' health subsidies and government intervention and then the equilibrium allocation with public subsidies and taxes.

4.1. *Laissez faire*

From budget constraints (2) and (3), the dynasty faces a constraint in each period as follows:

$$(13) \quad c_t = \frac{R_t s_{t-1}}{n_{t-1}} + \left(\frac{T(h_{t-1}, m_t)}{n_{t-1}} + 1 - v n_t \right) w_t - s_t - h_t - \frac{(d_t + m_t) T(h_{t-1}, m_t)}{n_{t-1}}.$$

The dynasty maximizes utility in (1) subject to (13), taking prices as given. The respective intergenerational and intertemporal substitution conditions are as follows:

$$(14) \quad \frac{\beta U'(T(h_{t-1}, m_t) d_t)}{\alpha U'(c_t)} = \frac{1}{n_{t-1}},$$

$$(15) \quad \frac{U'(c_t)}{\alpha U'(c_{t+1})} = \frac{R_{t+1}}{n_t}.$$

From $R_{t+1} = f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})$ in (6), conditions (14) and (15) are analogous to (8) and (9) chosen by the social planner. However, the health externalities can affect agents' intergenerational and intertemporal substitutions through $R_{t+1} = f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})$ as agents ignore the contribution of their health to the average productivity of the economy.

The first-order conditions with respect to health spending h_t and m_t are

$$(16) \quad \frac{T_h(h_t, m_{t+1}) \left[\beta U'(T(h_t, m_{t+1})d_{t+1})d_{t+1} + \frac{\alpha U'(c_{t+1})(w_{t+1} - d_{t+1} - m_{t+1})}{n_t} \right]}{U'(c_t)} = 1,$$

$$(17) \quad \frac{T_m(h_{t-1}, m_t) \left[\beta U'(T(h_{t-1}, m_t)d_t)d_t + \frac{\alpha U'(c_t)(w_t - d_t - m_t)}{n_{t-1}} \right]}{\frac{\alpha U'(c_t)T(h_{t-1}, m_t)}{n_{t-1}}} = 1.$$

Since perceived private returns to health spending are below the social returns, private health spending in (16) or (17) is below the optimal level in (10) or (11).

The first-order condition with respect to the number of children is

$$(18) \quad \frac{\rho G'(n_t)}{U'(c_t)} = v w_t + \frac{s_t}{n_t} + \frac{\alpha U'(c_{t+1})T(h_t, m_{t+1})(w_{t+1} - d_{t+1} - m_{t+1})}{U'(c_t)n_t^2},$$

where $w_t = f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)$ in (5). Low health spending caused by the health externalities leads to low average productivity and hence low wage rates or a low time cost of rearing a child $v w_t$ in (18) relative to the socially optimal levels. Low average productivity also means low returns to capital, thus leading to low savings in (15). The low savings lead to a low bequest cost of raising a child s_t/n_t , while the low health spending means low old-age earnings and parental transfers to children $T(h_t, m_{t+1})(w_{t+1} - d_{t+1} - m_{t+1})$ or a low transfer cost in (18). Thus, fertility is high and young-age labor is low relative to the socially optimal level in (12).

Since $s_t = n_t k_{t+1}$, the transversality condition associated with assets is the same as that for the social planner's allocation. Combining the transversality condition with successive substitutions on $\alpha u'(c_t) = u'(c_{t-1})n_{t-1}/R_t$ from condition (15) yields binding solvency

$$\lim_{t \rightarrow \infty} k_{t+1} \prod_{j=0}^t \frac{n_j}{R_j} = 0.$$

From the binding solvency, the gross returns to savings should exceed the growth rates of aggregate capital for dynamic efficiency and a bounded value function of the state when time approaches infinity.⁶

Definition 1. A competitive equilibrium from an initial state (h_{-1}, k_0, n_{-1}) is a sequence of allocations $\{b_t, c_t, d_t, h_t, m_t, n_t, s_t, l_t, k_{t+1}, y_t\}_{t=0}^{\infty}$ and prices $\{R_t, w_t\}_{t=0}^{\infty}$ such that: (i) given average health and prices, firms and families optimize, satisfying budget constraints (2) and

⁶ Otherwise, the marginal product of capital would be too low to compensate for capital depreciation, thus causing dynamic inefficiency, a scenario coined as over-savings ruled out by binding solvency in the dynastic model.

(3), technology (4), conditions (5), (6) and (14) to (18), the transversality condition, and binding solvency; (ii) all markets clear; (iii) consistency holds: $h = \bar{h}$ and $m = \bar{m}$.

The laissez-faire allocation resembles low health spending, longevity, savings and income but high fertility in developing countries. Next, we explore the roles of public subsidies and taxes in determining the equilibrium outcome and derive optimal taxes and subsidies.

4.2. Optimal public subsidies and taxes

Government spending includes a lump-sum transfer to the elderly P_t and subsidies on savings at rate ξ_t^S and on health spending at young and old age at respective rates ξ_t and ξ_t^O . Government revenue is from taxes on young- and old-age labor income at respective rates τ_t and τ_t^O , on consumption at rate τ_t^C , and on capital income $r_t = R_t - 1$ at rate τ_t^S , where $r_t > 0$ under the binding solvency for dynamic efficiency. The government balances its budget

$$\xi_t \bar{h}_t + \xi_t^S \bar{s}_t + \frac{\xi_t^O \bar{T}(\bar{h}_{t-1}, \bar{m}_t) \bar{m}_t + \bar{P}_t}{\bar{n}_{t-1}} = \tau_t (1 - v \bar{n}_t) w_t + \tau_t^C \bar{c}_t + \frac{(\tau_t^O w_t + \tau_t^C \bar{d}_t) \bar{T}(\bar{h}_{t-1}, \bar{m}_t) + \tau_t^S \bar{s}_{t-1} (R_t - 1)}{\bar{n}_{t-1}}.$$

Then, household budget constraints become

$$\begin{aligned} c_t (1 + \tau_t^C) &= b_t + (1 - v n_t) w_t (1 - \tau_t) - s_t (1 - \xi_t^S) - h_t (1 - \xi_t), \\ T(h_{t-1}, m_t) d_t (1 + \tau_t^C) &= T(h_{t-1}, m_t) w_t (1 - \tau_t^O) + [R_t - (R_t - 1) \tau_t^S] s_{t-1} \\ &\quad + P_t - T(h_{t-1}, m_t) m_t (1 - \xi_t^O) - b_t n_{t-1}. \end{aligned}$$

Combining household budget constraints into a single constraint for the dynasty yields

$$\begin{aligned} c_t &= \frac{(1 - v n_t) w_t (1 - \tau_t) - s_t (1 - \xi_t^S) - h_t (1 - \xi_t)}{(1 + \tau_t^C)} + \\ &\quad \frac{T(h_{t-1}, m_t) [w_t (1 - \tau_t^O) - m_t (1 - \xi_t^O) - d_t (1 + \tau_t^C)] + P_t + [R_t - (R_t - 1) \tau_t^S] s_{t-1}}{n_{t-1} (1 + \tau_t^C)}. \end{aligned}$$

The first-order condition with respect to inter-vivos transfers for intergenerational substitution is analogous to (14). The first-order condition with respect to savings s_t is

$$(19) \quad \frac{U'(c_t)}{\alpha U'(c_{t+1})} = \frac{(1 + \tau_t^C) [R_{t+1} - (R_{t+1} - 1) \tau_{t+1}^S]}{n_t (1 - \xi_t^S) (1 + \tau_{t+1}^C)},$$

in which the marginal rate of substitution between young-age consumption across generations equals the after-tax private return to savings (net of the subsidy) over the number of children. A rise in capital income tax τ_{t+1}^S reduces the relative price of current consumption, thus generating a negative substitution effect on savings. Conversely, a rise in the subsidy rate on savings ξ_t^S creates a positive substitution effect on savings. If consumption tax rates are constant $\tau_t^C = \tau_{t+1}^C$ to cancel out their wedge in the intertemporal substitution, then taxes on capital income τ_{t+1}^S and subsidies on savings ξ_t^S move in the same direction for optimal savings.

The first-order conditions with respect to health spending at young and old age equalize the private marginal benefits and costs of health spending, respectively, as follows:

$$(20) \quad \frac{T_h(h_t, m_{t+1}) \left[\beta U'(T(h_t, m_{t+1})d_{t+1})d_{t+1} + \frac{\alpha U'(c_{t+1})(w_{t+1}(1-\tau_{t+1}^o) - d_{t+1}(1+\tau_{t+1}^c) - m_{t+1}(1-\xi_{t+1}^o))}{n_t(1+\tau_{t+1}^c)} \right]}{\frac{U'(c_t)(1-\xi_t)}{(1+\tau_t^c)}} = 1,$$

$$(21) \quad \frac{T_m(h_{t-1}, m_t) \left[\beta U'(T(h_{t-1}, m_t)d_t)d_t + \frac{\alpha U'(c_t)(w_t(1-\tau_t^o) - d_t(1+\tau_t^c) - m_t(1-\xi_t^o))}{n_{t-1}(1+\tau_t^c)} \right]}{\frac{\alpha U'(c_t)T(h_{t-1}, m_t)(1-\xi_t^o)}{n_{t-1}(1+\tau_t^c)}} = 1.$$

Public health subsidies at young and old age ($\xi_t, \xi_t^o, \xi_{t+1}^o$) increase health spending by reducing the marginal costs of private health spending. By contrast, old-age labor income taxes (τ_t^o, τ_{t+1}^o) reduce the marginal benefits of health spending (hence inter-vivos transfers to children) by lowering the after-tax earnings at old age, and thus may lower private health spending. Consumption taxes (τ_t^c, τ_{t+1}^c) reduce the costs and benefits of health spending. If consumption tax rates are constant $\tau_t^c = \tau_{t+1}^c$, then their effects on health spending fully cancel out when substituting (14) into (20) and (21).

The first-order condition with respect to the number of children is

$$(22) \quad \frac{\rho G'(n_t)}{U'(c_t)} = \frac{vw_t(1-\tau_t)}{(1+\tau_t^c)} + \frac{s_t(1-\xi_t^s)}{n_t(1+\tau_t^c)} + \frac{\alpha U'(c_{t+1})P_{t+1}}{U'(c_t)n_t^2(1+\tau_{t+1}^c)} + \frac{\alpha U'(c_{t+1})T(h_t, m_{t+1})(w_{t+1}(1-\tau_{t+1}^o) - d_{t+1}(1+\tau_{t+1}^c) - m_{t+1}(1-\xi_{t+1}^o))}{U'(c_t)n_t^2(1+\tau_{t+1}^c)}.$$

Taxes on consumption τ_t^c and young-age labor income τ_t reduce the time cost of raising a child in (22), while taxes on old-age labor income reduce after-tax earnings at old age and the transfer cost of raising a child. A rise in savings subsidies lowers the bequest cost of raising a child, whereas a rise in public transfers to the elderly raises the transfer cost by increasing parental transfers to children. Moreover, a rise in old-age health subsidies decreases old-age health costs and increases inter-vivos transfers to children (the transfer cost of raising a child).

The government can design optimal taxes and subsidies by equating the equilibrium conditions with the optimal conditions of the social planner. Among the policy instruments actually used in developed countries, we can treat some instruments as given and use the other instruments to achieve the optimal outcome and balance the government budget.

Proposition 1. *Using the socially optimal allocation with $h = \bar{h}$ and $m = \bar{m}$ and taking public transfers to the elderly and taxes on consumption and capital income $\{P_t, \tau_t^c, \tau_t^s\}_{t=0}^\infty$ as given, the government can determine socially optimal health subsidies, savings subsidies and labor income taxes $\{\xi_t, \xi_t^o, \xi_t^s, \tau_t, \tau_t^o\}_{t=0}^\infty$ from the following conditions:*

$$(i) \quad \frac{vf_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)(\tau_t + \tau_t^c) + k_{t+1}(\tau_t^c + \xi_t^s)}{(1+\tau_t^c)} = \frac{P_{t+1}}{n_t f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})(1+\tau_{t+1}^c)}$$

$$\begin{aligned}
& + \frac{T(h_t, m_{t+1})[m_{t+1}(\xi_{t+1}^o + \tau_{t+1}^c) - f_l(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})(\tau_{t+1}^o + \tau_{t+1}^c)]}{n_t f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})(1 + \tau_{t+1}^c)}, \\
\text{(ii)} \quad f_{\bar{h}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) &= \frac{\xi_t + \tau_t^c}{1 + \tau_t^c} + \\
& \frac{T_h(h_t, m_{t+1})[m_{t+1}(\xi_{t+1}^o + \tau_{t+1}^c) - f_l(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})(\tau_{t+1}^o + \tau_{t+1}^c)]}{f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})(1 + \tau_{t+1}^c)}, \\
\text{(iii)} \quad f_{\bar{m}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) &= \frac{\xi_t^o T(h_{t-1}, m_t) + T_m(h_{t-1}, m_t)[m_t \xi_t^o - f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) \tau_t^o]}{n_{t-1}}, \\
\text{(iv)} \quad f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1}) &= \frac{\tau_{t+1}^s (1 + \tau_t^c)}{(\tau_{t+1}^c + \tau_{t+1}^s)(1 + \tau_t^c) - (\xi_t^s + \tau_t^c)(1 + \tau_{t+1}^c)}, \\
\text{(v)} \quad \xi_t h_t + \xi_t^s k_{t+1} n_t + \frac{\xi_t^o T(h_{t-1}, m_t) m_t + P_t}{n_{t-1}} &= f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) \left[\tau_t (1 - v n_t) + \right. \\
& \left. \frac{\tau_t^o T(h_{t-1}, m_t)}{n_{t-1}} \right] + \tau_t^c \left[c_t + \frac{d_t T(h_{t-1}, m_t)}{n_{t-1}} \right] + \tau_t^s k_t [f_k(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) - 1].
\end{aligned}$$

Proof. See Appendix A.

In condition (i) for the optimal policy, the positive effects of public transfers to the elderly or the old-age health subsidy on the costs of raising a child counteract the negative effects of labor income taxes, consumption taxes or savings subsidies on the costs of raising a child. Condition (ii) internalizes the marginal product of average young-age health spending by equating it with the young-age health subsidy plus the marginal contribution of young-age health spending to old-age labor via the old-age health subsidy net of the old-age labor income tax. Condition (iii) internalizes the marginal product of average old-age health spending by equating it with the old-age health subsidy plus the marginal contribution of old-age health spending to old-age labor via the old-age health subsidy net of the old-age labor income tax. Using (8) and (10) in condition (ii) under a stationary consumption tax rate $\tau_t^c = \tau_{t+1}^c$ yields:

$$f_{\bar{h}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t) = \xi_t + \frac{T_h(h_t, m_{t+1})[m_{t+1} \xi_{t+1}^o - f_l(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1}) \tau_{t+1}^o]}{f_k(k_{t+1}, \bar{\Omega}(\bar{h}_{t+1}, \bar{m}_{t+1}), l_{t+1})}.$$

From this and (iii), health subsidies are essential for internalizing the health externalities.

When consumption tax rates are constant, condition (iv) implies the same sign for subsidies on savings ξ_t^s and taxes on capital income τ_{t+1}^s to remove their wedge in intertemporal substitution. Condition (v) balances the government budget. To see the results more clearly, the next section focuses on the steady state with specific functional forms.

5. Steady state with specific functions

The production function is as follows:

$$(23) \quad y = Ak^\theta \left[\left(\bar{\Omega}(\bar{h}, \bar{m}) \right)^\mu l \right]^{1-\theta},$$

with $A > 0$, $0 < \theta, \mu < 1$, $\bar{\Omega}(\bar{h}, \bar{m}) = \bar{h}^\phi \bar{m}^{1-\phi}$, and $\phi \in (0,1)$, where μ measures the degree of health externalities to productivity and ϕ measures the relative role of average young-age health spending in determining the health externalities.

The utility function has a constant elasticity of intertemporal substitution for consumption and fertility, measured by $1/\sigma_1 > 0$ and $1/\sigma_2 > 0$, respectively, as follows:

$$(24) \quad U(x) = \frac{x^{1-\sigma_1}-1}{1-\sigma_1}, G(n) = \frac{n^{1-\sigma_2}-1}{1-\sigma_2},$$

where $x = c, T(h, m)d$. The longevity function is:

$$(25) \quad T(h, m) = D \left(\frac{z(h, m)}{\delta + \epsilon z(h, m)} \right)^\psi,$$

which increases with health spending at young and old age at diminishing rates under restrictions $D \in (0,1]$, $\delta > 0$, $\epsilon \geq 1$, $\psi \in (0,1)$, and $z(h, m) = h^\phi m^{1-\phi}$. In equilibrium, $z(h, m) = \bar{\Omega}(\bar{h}, \bar{m})$. Appendix B gives the socially optimal allocation at the steady state.

Using the specific functions, the rest of this section presents analytical results for optimal taxes and subsidies and then numerical results for quantitative implications.

5.1. Optimal public policies

We define the following expressions for the determination of optimal taxes and subsidies:

$$\begin{aligned} \Lambda_1 &\equiv \alpha \left[\frac{P}{n} + \frac{f_m(k, \Omega(h, m), l)}{T_m(h, m)} \left(T(h, m) + \frac{\alpha v n h T_h(h, m)}{\alpha(1-vn)-vn} \right) - \frac{v n h f_h(k, \Omega(h, m), l)}{\alpha(1-vn)-vn} \right], \\ \Lambda_2 &\equiv - \frac{(n-\alpha)(\alpha-vn)k}{\alpha(1-vn)-vn}, \\ \Lambda_3 &\equiv - \frac{\alpha \left\{ (1-vn)n[vf_l(k, \Omega(h, m), l) + k] - \frac{T(h, m)}{n} [v n d + \alpha(1-vn)(m - f_l(k, \Omega(h, m), l)) - v n c] \right\}}{\alpha(1-vn)-vn}, \\ \Lambda_4 &\equiv \left(\frac{\alpha T(h, m)}{n T_m(h, m)} \right) \left[T(h, m) + \frac{\alpha v n h T_h(h, m)}{\alpha(1-vn)-vn} \right], \\ \Lambda_5 &\equiv n[vf_l(k, \Omega(h, m), l) + k] - \alpha \left[c + \frac{T(h, m)(d+m-f_l(k, \Omega(h, m), l))}{n} \right]. \end{aligned}$$

The optimal subsidies and taxes in the steady state with the specific functions are as follows:

Proposition 2. *Using the social planner's allocation with the specific functions and taking public transfers to the elderly, consumption taxes and capital income taxes (P, τ^c, τ^s) as given, the government can determine socially optimal public health subsidies, savings subsidies and labor income taxes ($\xi, \xi^o, \xi^s, \tau, \tau^o$) in the steady state as follows:*

$$\begin{aligned} (i) \quad \xi^o &= \frac{\Lambda_1 + \Lambda_2 \tau^s + \Lambda_3 \tau^c}{\Lambda_4}, \\ (ii) \quad \xi &= (1-\alpha)f_h(k, \Omega(h, m), l) + \frac{\alpha T_h(h, m) T(h, m) \xi^o}{T_m(h, m) n}, \end{aligned}$$

$$\begin{aligned}
\text{(iii)} \quad \xi^s &= \frac{(n-\alpha)\tau^s}{n}, \\
\text{(iv)} \quad \tau^o &= \frac{\xi^o \left(m + \frac{T(h,m)}{T_m(h,m)} \right) - \frac{nf_m(k,\Omega(h,m),l)}{T_m(h,m)}}{f_l(k,\Omega(h,m),l)}, \\
\text{(v)} \quad \tau &= \frac{\alpha[h\xi + (n-\alpha)k\tau^s] + \Lambda_5\tau^c}{f_l(k,\Omega(h,m),l)[\alpha(1-vn) - vn]}.
\end{aligned}$$

Proof. See Appendix C.

Without health externalities ($\mu = 0$), $f_h(k, \Omega(h, m), l) = 0$ and $f_m(k, \Omega(h, m), l) = 0$ (see Appendix B). Then, setting all taxes, subsidies and public transfers to the elderly at zero yields socially optimal outcome. In other words, the laissez-faire allocation without any market friction would be the same as the planner's allocation in this lifecycle-dynastic model (the First Welfare Theorem). This feature is absent in lifecycle models with overlapping generations that abstract from intergenerational transfers. In the presence of health externalities to productivity $\mu > 0$, $f_h(k, \Omega(h, m), l)$ and $f_m(k, \Omega(h, m), l)$ are positive and increasing with μ (see Appendix B), and hence appropriate subsidies and taxes can internalize the health externalities.

In condition (iii), the optimal capital income tax τ^s and savings subsidy ξ^s share the same sign because dynamic efficiency or binding solvency requires $f_k(k, \Omega(h, m), l) = n/\alpha > 1$ (see Appendix B). The condition for dynamic efficiency or binding solvency implies that fertility must exceed the subjective discount factor $n > \alpha$. Intuitively, a high subjective discount factor leads to high savings (hence a low marginal product of capital), whereas high fertility leads to a high marginal product of capital. A high subjective discount factor may also lead to low fertility. This lower bound on fertility is novel in its own right.

Next, we focus on the quantitative implications for policy improvements.

5.2. Quantitative implications

Public health subsidies are available worldwide. Particularly, almost all the OECD countries have universal public healthcare such as Australia. In Figure 1, total and government health expenditures in Australia rose from 4.5% and 2.8% of GDP in 1971 to 10.5% and 7.6% in 2020, respectively, while male and female longevity at age 30 also rose from 0.37 and 0.56 in 1971 to 0.72 and 0.85 in 2017, respectively.⁷ In Figure 2, log real GDP per capita and the labor force participation of young agents at age 30–59 and old agents at age 60 and over increased together with a negative relationship with fertility as in other data (Bhargava et al., 2001; Weil, 2007; Au et al., 2005; Disney et al., 2006; Cai, 2010; Schultz, 2005).

⁷ As one period corresponds to 30 years in our model, longevity (T) is calculated from life expectancy at age 30 (LE) i.e., $T = LE/30 - 1$.

[Figures 1 and 2 go here]

Observations and calibration

We report the average values of taxes, subsidies, saving rates, fertility, life expectancy, real GDP per capita, total health spending (% of GDP), public health spending (% of GDP) in Australia during 2000–2020 (unless indicated otherwise) in Table 1 and benchmark parameterization in Table 2.

[Tables 1 and 2 go here]

From total health spending (8.8% of GDP) and public health spending (6% of GDP), the flat rate of health subsidies ($\xi = \xi^o$) is 68.18%. From the Australian tax system, the average labor income tax rates in young and old age ($\tau = \tau^o$) are 28%; the tax rate on returns to savings (τ^s) follows the 30% corporate income tax; and the consumption tax rate (τ^c) is 10%. As compulsory retirement savings (superannuation) are exemptible from labor income taxes, we set the savings subsidy rate (ξ^s) at 2%.⁸ We also set public transfers to the elderly at 20.6% of output per young worker (P/y) to balance the government budget.

Capital's income share takes a standard value $\theta = 0.33$. The degree of the health externalities at $\mu = 9\%$ follows the estimate of Bloom et al. (2022) that marginally better health of the labor force raises labor productivity by 6% to 12% in 133 countries when $\theta = 0.33$. Setting the coefficient of the health function ϵ at unity, the return factor on health spending ψ at 0.8 and the share of young-age health spending in the health technology ϕ at 0.5, we calibrate coefficients δ and D in the longevity function as well as total factor productivity A to match real GDP per capita, health spending (% of GDP) and old-age life T in data.

Intergenerational discount factor α at 0.7103 follows the savings rate at 22% when the taste for the old parent's consumption β is 0.3552, which is half of α because old-age life is shorter than the young age. The intertemporal elasticity of substitution for consumption $1/\sigma_1 = 1/1.1$ follows the estimates in the range 0.8 to 1 (Kydland and Prescott, 1982; Jones et al., 2000). Since a lower intertemporal elasticity of substitution for fertility than for consumption can account for the secular decline in fertility when income rises (e.g., Greenwood et al., 2005), we set $1/\sigma_2 = 1/1.5$. The fixed time rearing a child $v = 0.2$ matches observed fertility per young agent n at 0.91(= 1.82/2) and the young-age labor-force participation rate $1 - vn$ at 0.818, which is consistent with the labor-force participation rate of the population aged 30–59 in the

⁸ Superannuation contributions are 1.5% of GDP in Australia (<https://treasury.gov.au/speech/compulsory-superannuation-and-national-saving>). From the labor income tax rate at 28%, subsidized savings equal 0.42% of GDP. Then, the roundup savings subsidy is 2% as the national saving is 22% of GDP in the data (World Bank).

recent years in Australia. The taste for the number of children $\rho = 0.2769$ matches the fertility rate and other observations in Table 1.

Numerical results

Table 3 presents numerical results at steady states in three cases: laissez faire; benchmark; and socially optimum. Notably, the benchmark levels of old-age longevity or old-age labor and health spending at young and old age are higher than the laissez-faire levels due to public health spending in Australia but lower than the optimal levels, suggesting potential gains of increasing health subsidies to raise health spending toward the optimal level.

[Table 3 goes here]

The benchmark has lower fertility and higher young-age labor than the laissez-faire as benchmark health subsidies increase the costs of raising a child. The benchmark also has slightly lower fertility but higher young-age labor and old-age consumption than the optimal levels possibly due to low benchmark savings subsidies and high benchmark public transfers to the elderly that lead to a high cost of raising a child. The benchmark levels of capital and output per young worker and young-age consumption are higher than the laissez-faire levels but lower than the optimal levels. The benchmark welfare level at the steady state is higher than the laissez-faire level but lower than the optimal level.⁹

Table 4 reports taxes and subsidies in five cases. Case (1) is the benchmark. Cases (2) to (5) are socially optimal health subsidies with various taxes and public transfers to the elderly: benchmark taxes on consumption and capital income and public transfers to the elderly; a higher consumption tax; a lower public transfer to the elderly; or taxes on consumption and capital income without labor income tax and public transfers to the elderly.

[Table 4 goes here]

Optimal health subsidy rates ξ and ξ^o and old-age labor income tax rate τ^o in Case (2) are higher than the benchmark levels when public transfers to the elderly P/y and taxes on consumption τ^c and capital income τ^s are at the benchmark levels. The higher health subsidies increase health spending and the marginal cost of raising a child in (22), while the higher old-age labor income tax decreases the marginal benefit of health spending and the marginal cost of raising a child. The optimal subsidy on savings ξ^s is higher than the benchmark level to reduce the marginal cost of savings and the bequest cost of raising a child and increase savings

⁹ The small welfare gap is owing to the focus on the steady states across cases in the context of a modified golden rule. A full consideration of the equilibrium path with transitional dynamics is outside the scope of this study given the complexity of tracking down the entire dynamic path for welfare analysis.

and fertility to their optimal levels. The optimal young-age labor income τ is below the benchmark level, creating positive effects on the benefit of health spending and the time cost of rearing a child that offset the opposite effects from the rise in the old-age labor income tax.

A rise in the consumption tax to 20% in Case (3) raises optimal subsidies on health spending and savings and decreases optimal taxes on labor income at young and old age when capital income taxes and public transfers to the elderly remain at the benchmark levels. The rise in the consumption tax reduces the marginal cost of raising a child. A fall in public transfers to the elderly to 15% of GDP per young worker in Case (4) with benchmark taxes on consumption and capital income has similar effects on optimal labor income taxes and health subsidies as those in Case (3) because the fall in public transfers to the elderly reduces the transfer cost of raising a child. The removal of labor income taxes and public transfers to the elderly in Case (5) increases the optimal health subsidy to young workers and the optimal consumption tax, decreases the optimal health subsidy to old workers, and leads to an optimal tax on savings but an optimal subsidy on capital income. The tax on savings increases the bequest cost of raising a child, while the rise in consumption taxes reduces the cost of raising a child.

Figure 3 illustrates the sensitivity of the optimal allocation, health subsidy rates, savings subsidy rates, and labor income tax rates at the steady state to variations in the degree of health externalities to productivity when holding other policy instruments at benchmark levels. Internalizing a stronger health externality (higher μ) increases the optimal levels of consumption, capital per young worker, output per young worker, health spending, longevity, and productivity. However, the effect of higher μ on optimal fertility is U-shaped, while the effect on optimal young-age labor is inverted U-shaped, mainly due to the diminishing positive effect of health externalities on productivity. Intuitively, internalizing a stronger health externality to productivity has a positive income effect and a negative substitution effect on fertility. At low μ , internalizing a stronger health externality has strong effects on productivity or the time cost of rearing a child, and thus the negative substitution effect on fertility dominates. At high μ , the effect on productivity diminishes, and thus the positive income effect on fertility dominates. The internalization of higher μ improves social welfare.

Internalizing a stronger health externality increases optimal health subsidy rates and young-age labor income tax rates but decreases optimal old-age labor income tax rates until the externality is sufficiently strong. The effect of higher μ on optimal savings subsidy rates is also U-shaped: initially negative at low longevity and eventually positive at high longevity. The U-

shaped pattern for optimal savings subsidy rates helps obtain the U-shaped pattern of optimal fertility since savings subsidies reduce the bequest cost of raising a child.

[Figure 3 goes here]

Next, we explore firms' health subsidies for profit maximization.

6. Firms' health subsidies

Firms' health subsidies in terms of employer-based health insurance (EHI) are popular in the US where public healthcare only covers 35% of population (OECD, 2016). According to US Bureau statistics, EHI covers almost 85% of the population with private health insurance. EHI has the advantage of group insurance over an individual's health insurance to ease the concern of private information on an individual's health status or health spending.

Let $\pi_t \in (0,1)$ and $\pi_t^o \in (0,1)$ denote the portions of young- and old-age health spending covered by health insurance, respectively. In equilibrium with a competitive insurance market, health insurance premium per young or old worker I_t or I_t^o equals the insurance coverage of health spending per young worker, $I_t = \bar{h}_t \pi_t$, or per old worker, $I_t^o = \bar{T}(\bar{h}_{t-1}, \bar{m}_t) \bar{m}_t \pi_t^o$.

With EHI, we rewrite the production function as

$$(26) \quad y_t = f\left(k_t, \bar{\Omega}\left(I_t + \bar{H}_t, \frac{I_t^o + \bar{M}_t}{\bar{T}(\bar{h}_{t-1}, \bar{m}_t)}\right), l_t\right),$$

where $\bar{H}_t \equiv \bar{h}_t(1 - \pi_t)$ is average out-of-pocket health spending per young worker, and $\bar{M}_t \equiv \bar{T}(\bar{h}_{t-1}, \bar{m}_t) \bar{m}_t(1 - \pi_t^o)$ is average out-of-pocket health spending per old worker. Firms observe health insurance premium (I_t, I_t^o) but do not observe an individual's health spending (h_t, m_t) and old-age longevity $T(h_{t-1}, m_t)$.

Firms' profit function becomes

$$f\left(k_t, \bar{\Omega}\left(I_t + \bar{H}_t, \frac{I_t^o + \bar{M}_t}{\bar{T}(\bar{h}_{t-1}, \bar{m}_t)}\right), l_t\right) - R_t k_t - w_t l_t - \lambda_t I_t - \lambda_t^o I_t^o / n_{t-1},$$

where $\lambda_t \in (0,1)$ and $\lambda_t^o \in (0,1)$ are the rates of firms' subsidies on young-age insurance I_t and old-age insurance per young worker I_t^o / n_{t-1} , respectively. Profit maximization yields

$$(27) \quad w_t = f_l\left(k_t, \bar{\Omega}\left(I_t + \bar{H}_t, \frac{I_t^o + \bar{M}_t}{\bar{T}(\bar{h}_{t-1}, \bar{m}_t)}\right), l_t\right) = f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t),$$

$$(28) \quad R_t = f_k\left(k_t, \bar{\Omega}\left(I_t + \bar{H}_t, \frac{I_t^o + \bar{M}_t}{\bar{T}(\bar{h}_{t-1}, \bar{m}_t)}\right), l_t\right) = f_k(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t),$$

$$(29) \quad \lambda_t = f_l\left(k_t, \bar{\Omega}\left(I_t + \bar{H}_t, \frac{I_t^o + \bar{M}_t}{\bar{T}(\bar{h}_{t-1}, \bar{m}_t)}\right), l_t\right) = f_{\bar{h}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t),$$

$$(30) \quad \frac{\lambda_t^o}{n_{t-1}} = f_{I^o}\left(k_t, \bar{\Omega}\left(I_t + \bar{H}_t, \frac{I_t^o + \bar{M}_t}{\bar{T}(\bar{h}_{t-1}, \bar{m}_t)}\right), l_t\right) = \frac{f_{\bar{m}}(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)}{\bar{T}(\bar{h}_{t-1}, \bar{m}_t)}.$$

In (27) and (28), the marginal products or rental prices depend on EHI. In (29) and (30), the rates of firms' subsidies on health insurance for young and old workers depend positively on the marginal products of average health spending by young and old workers, respectively.

In what follows, we first focus on an economy with EHI in the absence of government intervention. Then, we consider both EHI and public policies.

6.1. Equilibrium allocations with EHI

With EHI, the budget constraint of a young worker is

$$(31) \quad c_t = b_t + (1 - vn_t)w_t - h_t(1 - \pi_t) - I_t(1 - \lambda_t) - s_t,$$

where $h_t(1 - \pi_t)$ is out-of-pocket health spending at young age and $I_t(1 - \lambda_t)$ the portion of health insurance premium paid by a young worker. The budget constraint of an old worker is

$$(32) \quad d_t T(h_{t-1}, m_t) = T(h_{t-1}, m_t)w_t + R_t s_{t-1} - m_t T(h_{t-1}, m_t)(1 - \pi_t^o) - I_t^o(1 - \lambda_t^o) - b_t n_{t-1},$$

where $m_t T(h_{t-1}, m_t)(1 - \pi_t^o)$ is out-of-pocket health spending at old age and $I_t^o(1 - \lambda_t^o)$ is the portion of health insurance premium paid by the old worker.

The dynasty maximizes utility subject to household budget constraints, taking prices, $\pi_t, I_t, \lambda_t, \pi_t^o, I_t^o$ and λ_t^o as given. The first-order conditions with respect to d_t and s_t are similar to (14) and (15) in laissez faire; and those with respect to h_t, m_t and n_t are

$$(33) \quad \frac{T_h(h_t, m_{t+1}) \left[\beta U'(T(h_t, m_{t+1})d_{t+1})d_{t+1} + \frac{\alpha U'(c_{t+1})(w_{t+1} - d_{t+1} - m_{t+1}(1 - \pi_{t+1}^o))}{n_t} \right]}{U'(c_t)(1 - \pi_t)} = 1,$$

$$(34) \quad \frac{T_m(h_{t-1}, m_t) \left[\beta U'(T(h_{t-1}, m_t)d_t)d_t + \frac{\alpha U'(c_t)(w_t - d_t - m_t(1 - \pi_t^o))}{n_{t-1}} \right]}{\frac{\alpha U'(c_t)T(h_{t-1}, m_t)(1 - \pi_t^o)}{n_{t-1}}} = 1,$$

$$(35) \quad \frac{\rho G'(n_t)}{U'(c_t)} = v w_t + \frac{s_t}{n_t} + \frac{\alpha U'(c_{t+1})[T(h_t, m_{t+1})(w_{t+1} - d_{t+1} - m_{t+1}) + I_{t+1}^o \lambda_{t+1}^o]}{U'(c_t)n_t^2}.$$

In equilibrium, $w_t = f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)$, $\bar{h}_{t-1} = h_{t-1}$, $\bar{m}_t = m_t$ and $\bar{T}(\bar{h}_{t-1}, \bar{m}_t) = T(h_{t-1}, m_t)$. In (33) and (34), the insurance coverage rates on young- and old-age health spending ($\pi_t, \pi_t^o, \pi_{t+1}^o$) reduce the marginal costs of health spending for young and old agents, thereby tending to raise young- and old-age health spending. However, firms' subsidy on old-age health insurance $I_{t+1}^o \lambda_{t+1}^o$ increases the transfer cost of raising a child in (35), tending to reduce fertility from its socially optimal level in (12). As a result, EHI alone cannot fully internalize health externalities to productivity when individuals choose the number of children.

6.2. Optimal EHI and optimal public policies

Household budget constraints with EHI and public policies become

$$\begin{aligned}
c_t(1 + \tau_t^c) &= b_t + (1 - vn_t)w_t(1 - \tau_t) - s_t(1 - \xi_t^s) - h_t(1 - \pi_t - \xi_t) - I_t(1 - \lambda_t), \\
T(h_{t-1}, m_t)d_t(1 + \tau_t^c) &= T(h_{t-1}, m_t)w_t(1 - \tau_t^o) + [R_t - (R_t - 1)\tau_t^s]s_{t-1} + P_t \\
&\quad - T(h_{t-1}, m_t)m_t(1 - \pi_t^o - \xi_t^o) - I_t^o(1 - \lambda_t^o) - b_t n_{t-1}.
\end{aligned}$$

Combining these budget constraints into a single constraint for the dynasty yields

$$(36) \quad c_t = \frac{(1 - vn_t)w_t(1 - \tau_t) - s_t(1 - \xi_t^s) - h_t(1 - \pi_t - \xi_t) - I_t(1 - \lambda_t)}{(1 + \tau_t^c)} + \frac{T(h_{t-1}, m_t)[w_t(1 - \tau_t^o) - m_t(1 - \pi_t^o - \xi_t^o) - d_t(1 + \tau_t^c)] + P_t + [R_t - (R_t - 1)\tau_t^s]s_{t-1} - I_t^o(1 - \lambda_t^o)}{n_{t-1}(1 + \tau_t^c)}.$$

The first-order conditions with respect to d_t and s_t are similar to (14) and (19); and those with respect to h_t , m_t and n_t are

$$(37) \quad \frac{T_h(h_t, m_{t+1})\beta U'(T(h_t, m_{t+1})d_{t+1})d_{t+1}}{\frac{U'(c_t)(1 - \pi_t - \xi_t)}{(1 + \tau_t^c)}} + \frac{T_h(h_t, m_{t+1})\left[\frac{\alpha U'(c_{t+1})(w_{t+1}(1 - \tau_{t+1}^o) - d_{t+1}(1 + \tau_{t+1}^c) - m_{t+1}(1 - \pi_{t+1}^o - \xi_{t+1}^o))}{n_t(1 + \tau_{t+1}^c)}\right]}{\frac{U'(c_t)(1 - \pi_t - \xi_t)}{(1 + \tau_t^c)}} = 1,$$

$$(38) \quad \frac{T_m(h_{t-1}, m_t)\left[\beta U'(T(h_{t-1}, m_t)d_t)d_t + \frac{\alpha U'(c_t)(w_t(1 - \tau_t^o) - d_t(1 + \tau_t^c) - m_t(1 - \pi_t^o - \xi_t^o))}{n_{t-1}(1 + \tau_t^c)}\right]}{\frac{\alpha U'(c_t)T(h_{t-1}, m_t)(1 - \pi_t^o - \xi_t^o)}{n_{t-1}(1 + \tau_t^c)}} = 1,$$

$$(39) \quad \frac{\rho G'(n_t)}{U'(c_t)} = \frac{vw_t(1 - \tau_t)}{(1 + \tau_t^c)} + \frac{s_t(1 - \xi_t^s)}{n_t(1 + \tau_t^c)} + \frac{\alpha U'(c_{t+1})P_{t+1}}{U'(c_t)n_t^2(1 + \tau_{t+1}^c)} + \frac{\alpha U'(c_{t+1})I_{t+1}^o\lambda_{t+1}^o}{U'(c_t)n_t^2(1 + \tau_{t+1}^c)} + \frac{\alpha U'(c_{t+1})T(h_t, m_{t+1})[w_{t+1}(1 - \tau_{t+1}^o) - d_{t+1}(1 + \tau_{t+1}^c) - m_{t+1}(1 - \xi_{t+1}^o)]}{U'(c_t)n_t^2(1 + \tau_{t+1}^c)},$$

where $w_t = f_l(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)$ and $R_t = f_k(k_t, \bar{\Omega}(\bar{h}_t, \bar{m}_t), l_t)$.

In (37) and (38), insurance coverage rates on health spending (π_t , π_t^o , π_{t+1}^o) are substitutes of public health subsidies (ξ_t , ξ_t^o , ξ_{t+1}^o) in raising health spending by reducing the marginal costs of health spending for young and old agents. In (39), firms' health subsidy $I_{t+1}^o\lambda_{t+1}^o$ increases the transfer cost of raising a child. Thus, the government can set appropriate taxes with the opposite effects on the cost of raising a child to that of firms' health subsidy to correct excessive fertility in the laissez-faire equilibrium for its socially optimal level. Next, we show optimal EHI and taxes in the steady state with the specific functions given in Section 5.

6.3. Steady-state analyses

We define the following expressions for efficient policies:

$$\Lambda_6 \equiv \frac{T(h, m)d}{n} + c + (1 - vn) \left[\frac{T(h, m)\alpha(m - f_l(k, \Omega(h, m), l))}{vn^2} - f_l(k, \Omega(h, m), l) - \frac{k}{v} \right],$$

$$\Lambda_7 \equiv \frac{(1 - vn)\alpha m f_m(k, \Omega(h, m), l)}{vn} + \frac{mT(h, m)[vn - \alpha(1 - vn)]}{vn^2},$$

$$\Lambda_8 \equiv f_l(k, \Omega(h, m), l) \left[\frac{T(h, m)(vn - \alpha(1 - vn))}{vn^2} + \frac{\alpha m(1 - vn)f_m(k, \Omega(h, m), l)T_m(h, m)}{vn(T_m(h, m)m + T)} \right].$$

The efficient policies with the specific functions in the steady state are as follows:

Proposition 3. *Using the social planner's allocation with the specific functions, the socially optimal public policies and firms' health subsidy rates $(\pi, \pi^o, I, I^o, \lambda, \lambda^o)$ in the steady state arise from the conditions for utility and profit maximization under feasibility and a balanced government budget.*

Proof. See Appendix D.

Taking public health subsidies, public transfers to the elderly, consumption taxes and capital income taxes $(\xi, \xi^o, P, \tau^c, \tau^s)$ as given, stronger externalities of health spending at old or young age raise the optimal values for health insurance coverage, insurance premium, and firms' health subsidies $(\pi, \pi^o, I, I^o, \lambda, \lambda^o)$ in Appendix D. Moreover, optimal insurance coverage rates (π, π^o) and public health subsidies (ξ, ξ^o) are perfect substitutes, suggesting a key role of EHI in easing the financial pressure of funding public healthcare.

Using parameter values in Table 2, we report numerical results for socially optimal policies in Tables 5 and 6 and their sensitivity to variations in the degree of health externalities μ in Figure 4. EHI in Columns (1) to (3) of Table 5 attains the social optimum at benchmark $\tau^c, \tau^s, P/y, \xi$ and ξ^o or at optimal income taxes even without public health subsidies in contrast to the results in Table 4. Notably, the optimal rate of firms' subsidy is higher on young-age health insurance than on old-age health insurance ($\lambda > \lambda^o$). In the US, employers often subsidize health insurance for workers, while the elderly (at 65 or over) may have access to public healthcare. In Table 6 at benchmark τ^c, τ^s and P/y , declines in public health subsidies raise the optimal values of health insurance coverage rates (π, π^o) , health insurance premium (I, I^o) and old-age labor income tax τ^o , but lower the optimal young-age labor income tax τ .

[Tables 5 and 6 go here]

Figure 4 shows the sensitivity of optimal EHI and labor income taxes to variations in the degree of health externalities μ at benchmark τ^c, τ^s and P/y in the absence of public health subsidies at the steady state. When μ rises, optimal rates for $\pi, \pi^o, I/y, I^o/y, \lambda$ and λ^o rise. However, when μ rises, the optimal old-age labor income tax rate τ^o rises but the optimal young-age labor income tax rate τ falls in contrast to falling optimal τ^o and rising optimal τ to finance rising optimal public health subsidies in Figure 3.

[Figure 4 goes here]

7. Conclusion

This paper developed a lifecycle-dynastic model of health spending, savings, and fertility to explore the effects of health externalities to productivity for policy implications. The health

externalities cause sub-optimally low health spending, longevity, savings, labor, and productivity but high fertility. Appropriate taxes and subsidies can attain the socially optimal outcome. Public health subsidies have a positive effect on the cost of raising a child and a negative effect on the cost of health spending. Conversely, labor income taxes have negative effects on the costs of raising a child and the benefits of health spending. Consumption taxes and savings subsidies have negative effects on the costs of raising a child. Savings subsidies also have a negative effect on the cost of savings that counteracts the wedge of capital income taxes. Public transfers to the elderly increase tax burdens on children and induce altruistic parents to increase transfers to children, thus increasing the transfer cost of raising a child.

From calibration results based on the Australian taxes and subsidies, fertility is below the laissez-faire and socially optimal levels, while labor supply at young age is above the laissez-faire and socially optimal levels. The levels of longevity, capital per young worker, output per young worker, young-age consumption, young- and old-age health spending and old-age labor are above the laissez-faire levels but below the socially optimal levels. In Australia, Medicare provides a universal health insurance coverage funded by taxes, including an income-tested Medicare levy surcharge at 1% to 1.5% of income. The quantitative implications of the results suggest policy improvements from the Australian tax system and public healthcare, such as increasing young-age health subsidies and consumption taxes or decreasing young- and old-age labor income taxes and public transfers to the elderly. The optimal tax here departs from those in models with fixed fertility and without health externalities to productivity. It relaxes the strict equality between the rates of capital income taxes and investment subsidies in the Ramsey tax. It also allows for age-dependent health subsidies and labor income taxes in relation to the age-dependent marginal products of health spending at young and old age.

Finally, employer-based health insurance reduces the marginal costs of health spending for young and old workers, thus raising young- and old-age health spending to internalize the health externalities. However, the subsequent improvement in productivity raises the time cost of raising a child. Additionally, firms' subsidy on old-age health insurance increases the transfer cost of raising a child. Thus, firms' health subsidies for profit maximization alone cannot fully internalize health externalities to productivity when individuals choose the number of children. Appropriate mixes of firms' health subsidies and efficient tax policies can attain the social optimum. From the results, expanding public health coverage to all workers can increase social welfare in the US, whereas encouraging employer-based health insurance can ease the financial pressure of universal public healthcare in other OECD countries.

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TABLE 1. Selected observations in Australia

Descriptions	Variables	Value
Savings subsidy	ξ^s	2%
Consumption tax	τ^c	10%
Old- and young-age labor income tax	$\tau^o = \tau$	28%
Capital income tax	τ^s	30%
Old-age public transfers (% of GDP)	P/y	20.60%
Old- and young-age health subsidies (% of GDP)	$\xi^o = \xi$	68.18%
Total health spending (% of GDP)	$(m + h)/y$	8.8%
Longevity at age 30	T	0.7522
Savings rate (% of GDP)	s/y	22%
Fertility per young parent	n	0.91
Real GDP per capita	y	53308

SOURCE: Consumption tax (The Australian Treasury, 2015; OECD, 2020); taxes on labor and capital income (OECD, 2000-2020); longevity (Australian Government Actuary, 2000-2017); total and public health spending % of GDP (OECD, 2000-2020); fertility (OECD, 2000-2020); savings rate and real GDP per capita (World Bank, 2000-2020).

TABLE 2. Benchmark parameterization

$t = 30$	Number of years per period
$v = 0.2$	Fixed time rearing a child
$\alpha = 0.7103$	Intergenerational discounting factor
$\beta = 0.3552$	Taste for parental old-age consumption
$\rho = 0.2769$	Taste for the number of children
$\sigma_1 = 1.1$	Reciprocal of intertemporal elasticity of substitution for consumption
$\sigma_2 = 1.5$	Reciprocal of intertemporal elasticity of substitution for fertility
$\theta = 0.33$	Capital's income share
$A = 1055.2795$	Total factor productivity
$\delta = 431.3664$	Coefficient reducing the effectiveness of health spending
$\epsilon = 1$	Coefficient reducing the marginal effectiveness of health spending
$\psi = 0.8$	Return factor on health spending
$D = 0.8648$	Autonomous factor of longevity
$\phi = 0.5$	Share of young-age health spending in the technology of health status
$\mu = 0.09$	Elasticity of output with respect to average health in final goods production

TABLE 3. Comparisons of numerical results at steady states

	(1)	(2)	(3)
Variables	Laissez-faire	Australia Benchmark	Social Optimum
Old-age longevity (T)	0.6856	0.7522	0.7672
Young-age health spending (h)	887.66	1735.4	2308.9
Old-age health spending (m)	1847.9	2955.7	3095.2
Capital intensity (k)	10264	12888	14053
Output per young worker (y)	44390	53308	55417
Fertility (n)	1.0137	0.9100	0.9244
Young-age labor ($1 - \nu n$)	0.7973	0.8180	0.8151
Young-age consumption (c)	20790	24333	24441
Old-age consumption (d)	16350	15811	15793
Welfare (WF)	22.891	22.987	23.011

TABLE 4. Numerical results on taxes and subsidies (%) at steady states

	(1)	(2)	(3)	(4)	(5)
Taxes and subsidies	Australia Benchmark	Optimal rates at benchmark $\tau^c, \tau^s, P/y$	Optimal rates at $\tau^c = 20\%$	Optimal rates at $\frac{P}{y} = 15\%$	Optimal rates at $\tau^o = \tau = P/y = 0$
ξ	0.6818	0.8195	0.7496	0.7631	0.6966
ξ^o	0.6818	0.7716	0.6832	0.7003	0.6161
ξ^s	0.0200	0.0695	0.0695	0.0695	-0.1464
τ	0.2800	0.2453	0.2546	0.2349	0.0000
τ^o	0.2800	0.4052	0.1748	0.2194	0.0000
τ^c	0.1000	0.1000	0.2000	0.1000	0.1056
τ^s	0.3000	0.3000	0.3000	0.3000	-0.6322
P/y	0.2060	0.2060	0.2060	0.1500	0.0000

TABLE 5. Optimal public policies and health insurance at steady states

	(1)	(2)	(3)
Policies	Optimal rates at benchmark $\xi, \xi^o, \tau^c, \tau^s, P/y$	Optimal rates at $\xi = \xi^o =$ $\tau^c = \tau^s = \frac{P}{y} = 0$	Optimal rates at $\xi = \xi^o =$ $\tau^c = \tau^o = \tau = \frac{P}{y} = 0$
π	0.1401	0.7224	0.6966
π^o	0.0928	0.6487	0.6161
I/y	0.0058	0.0301	0.0290
I^o/y	0.0040	0.0278	0.0264
τ	0.2076	-0.0866	0.0000
τ^o	0.4130	0.0850	0.0000
ξ^s	0.0695	0.0000	0.0563
ξ	0.6818	0.0000	0.0000
ξ^o	0.6818	0.0000	0.0000
τ^c	0.1000	0.0000	0.0000
τ^s	0.3000	0.0000	0.2431
P/y	0.2060	0.0000	0.0000

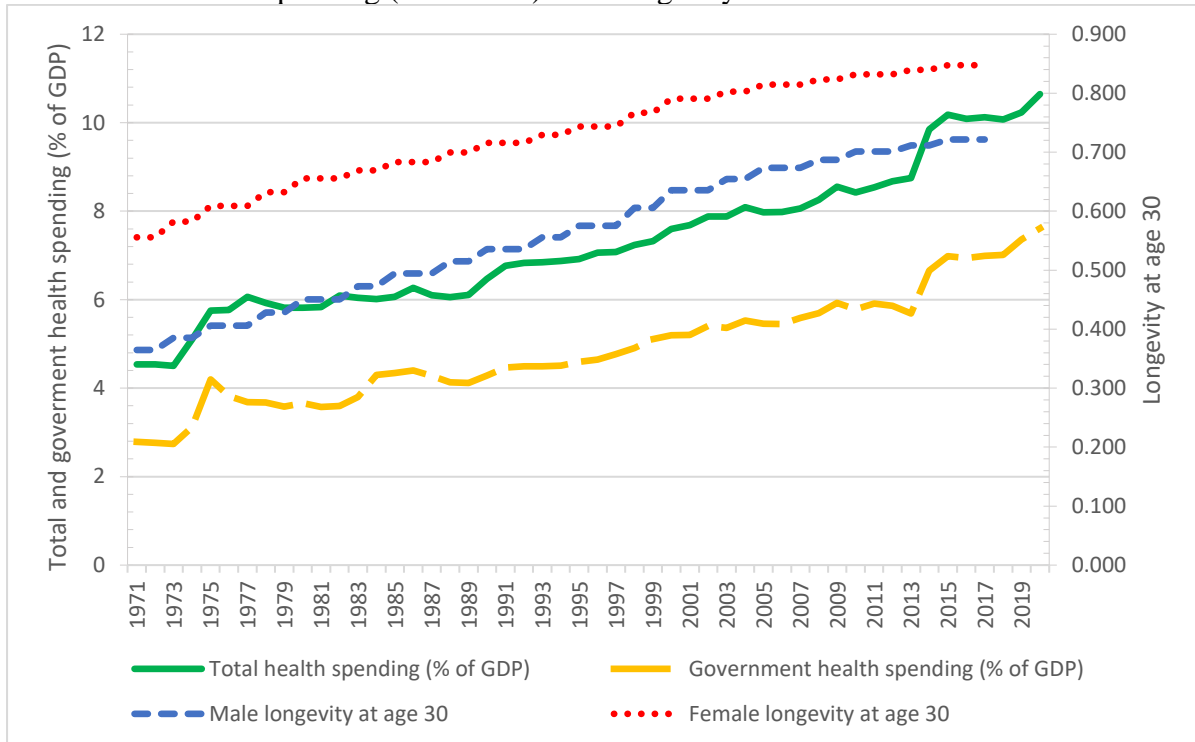
Optimal $\lambda = 0.7236$ and $\lambda^o = 0.6503$ are fixed across cases as they do not vary with public policies.

TABLE 6. Optimal health insurance and labor income taxes at benchmark τ^c, τ^s and P/y at steady state

	(1)	(2)	(3)	(4)
Policies	Optimal rates given $\xi = \xi^o = 0.6$	Optimal rates given $\xi = \xi^o = 0.4$	Optimal rates given $\xi = \xi^o = 0.2$	Optimal rates given $\xi = \xi^o = 0$
π	0.2232	0.4265	0.6298	0.8331
π^o	0.1763	0.3805	0.5847	0.7889
I/y	0.0093	0.0178	0.0262	0.0347
I^o/y	0.0076	0.0163	0.0251	0.0338
τ	0.1813	0.1172	0.0531	-0.0110
τ^o	0.4174	0.4283	0.4392	0.4501

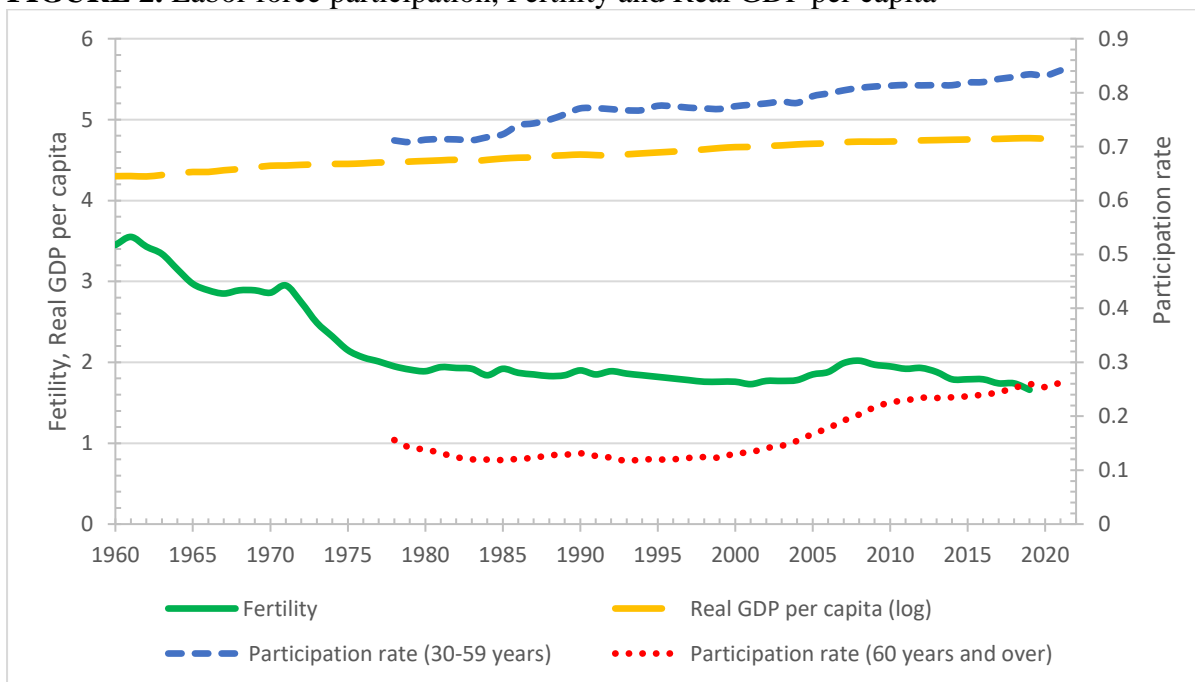
Optimal $\xi^s = 0.0695$ as it does not vary with ξ and ξ^o .

FIGURE 1. Health spending (% of GDP) and Longevity



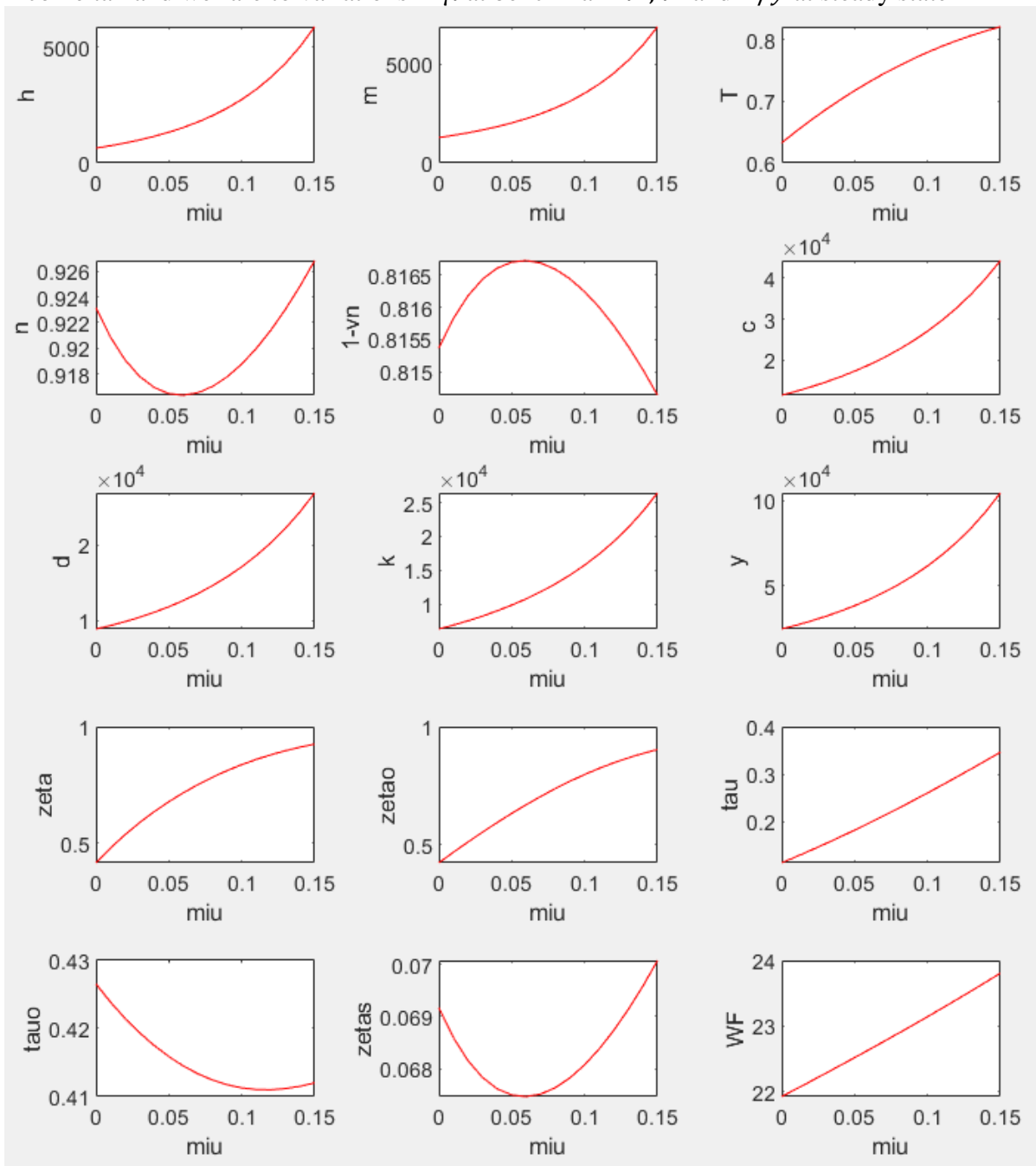
NOTE: Longevity at age 30, T , is calculated from life expectancy at age 30, LE , i.e., $T = LE/30 - 1$, where LE is from Australian Government Actuary. For example, the average male life expectancy at age 30, LE , in year 2015–2017 is around 51.6, so average male longevity at age 30 is $T = 51.6/30 - 1 = 0.72$. Total and public health spending as % of GDP is from OECD.

FIGURE 2. Labor force participation, Fertility and Real GDP per capita



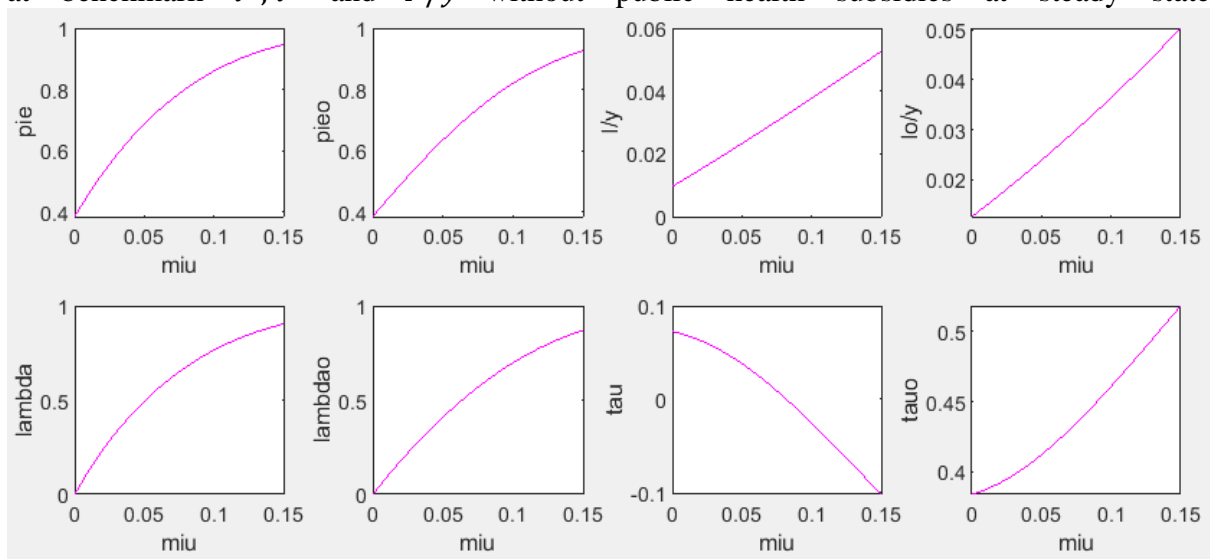
SOURCE: Labor force participation rate (Australian Bureau of Statistics); Fertility (OECD); Real GDP per capita (World Bank).

FIGURE 3. Sensitivity of optimal allocations, public health and savings subsidies, labor income tax and welfare to variations in μ at benchmark τ^c, τ^s and P/y at steady state



NOTE: zeta is ξ , zetao is ξ^o , tau is τ , tauo is τ^o , zetas is ξ^s , and WF is welfare.

FIGURE 4. Sensitivity of optimal health insurance and labor income taxes to variations in μ at benchmark τ^c, τ^s and P/y without public health subsidies at steady state



NOTE: pie is π , pieo is π^o , Io/y is I^o/y , lambda is λ , lambdao is λ^o , and tauo is τ^o .

Appendixes

A. Proof of Proposition 1. These conditions for socially optimal taxes and subsidies arise from equating the equilibrium conditions with the socially optimal conditions. Substituting market clearing conditions, consistency, zero profit condition, (5), (6) and (v) into the single budget constraint of the dynasty recovers feasibility in (7). Substituting consistency, (i), (iv), (5), (6), (19) and $s_t = k_{t+1}n_t$ into (22) yields (12). Substituting consistency, (ii), (iv), (5), (6) and (19) into (20) yields (10). Substituting consistency, (iii), (5), (6) and (14) into (21) attains (11). Substituting (iv), (5) and (6) into (19) gives (9).

B. Steady-state socially optimal allocation with specific functions. Using specific functions, the steady-state socially optimal allocation from conditions (7) to (12) is as follows:

$$(A1) \quad c = \frac{y(1-\alpha\theta)-h-mT(h,m)/n}{1+(1/n)(\beta n/\alpha)^{\frac{1}{\sigma_1}}},$$

$$(A2) \quad d = \left(\frac{\beta n}{\alpha}\right)^{\frac{1}{\sigma_1}} \left(\frac{c}{T(h,m)}\right),$$

$$(A3) \quad k = \alpha\theta y/n,$$

$$(A4) \quad f_h(k, \Omega(h, m), l) + \left(\frac{\alpha}{n}\right) T_h(h, m)[f_l(k, \Omega(h, m), l) - m] - 1 = 0,$$

$$(A5) \quad f_m(k, \Omega(h, m), l) + \left(\frac{T_m(h, m)}{n}\right) [f_l(k, \Omega(h, m), l) - m] - \frac{T(h, m)}{n} = 0,$$

$$(A6) \quad \rho n^{2-\sigma_2} c^{\sigma_1} + \alpha \left[c \left(\frac{\beta n}{\alpha}\right)^{\frac{1}{\sigma_1}} + mT(h, m) \right] - f_l(k, \Omega(h, m), l)[vn^2 + \alpha T(h, m)] + y\alpha\theta n = 0,$$

$$\text{where } y = \left[A \left(\frac{\alpha\theta}{n}\right)^\theta \right]^{\frac{1}{1-\theta}} (\Omega(h, m))^\mu l, \quad f_l(k, \Omega(h, m), l) = (1-\theta)y/l,$$

$$f_k(k, \Omega(h, m), l) = \theta y/k, \quad T_h(h, m) = \frac{T(h, m)\psi\delta\phi}{[\delta+\epsilon z(h, m)]h}, \quad T_m(h, m) = \frac{T(h, m)\psi\delta(1-\phi)}{[\delta+\epsilon z(h, m)]m},$$

$$f_h(k, \Omega(h, m), l) = \mu(1-\theta)y\phi/h, \quad f_m(k, \Omega(h, m), l) = \mu(1-\theta)y(1-\phi)/m.$$

From (A3) and the expression for f_k , binding solvency requires $R = f_k = n/\alpha > 1$.

C. Proof of Proposition 2. Conditions (19) to (22) with specific functions at steady state are

$$(A7) \quad s = \frac{\alpha\theta yn(1-\tau^s)}{n(1-\xi^s)-\alpha\tau^s},$$

$$(A8) \quad \left(\frac{\alpha}{n}\right) T_h(h, m)[(1-\tau^o)f_l(k, \Omega(h, m), l) - (1-\xi^o)m] - (1-\xi) = 0,$$

$$(A9) \quad T_m(h, m)[(1-\tau^o)f_l(k, \Omega(h, m), l) - (1-\xi^o)m] - (1-\xi^o)T(h, m) = 0,$$

$$(A10) \quad \rho n^{2-\sigma_2} c^{\sigma_1} (1 + \tau^c) - f_l(k, \Omega(h, m), l)[(1-\tau)vn^2 + \alpha(1-\tau^o)T(h, m)] + \alpha \left[c \left(\frac{\beta n}{\alpha}\right)^{\frac{1}{\sigma_1}} (1 + \tau^c) + (1-\xi^o)mT(h, m) - P - \theta yn + \tau^s(\theta yn - 1) \right] = 0.$$

Substituting (iii) into (A7) yields (A3). Substituting (ii) and (iv) into (A8) attains (A4). Substituting (iv) into (A9) yields (A5). Substituting (i), (ii), (iv) and (v) into (A10) gives (A6). From (s, h, m, n) in conditions (A7) to (A10), other variables follow Appendix B.

D. Proof of Proposition 3. At the steady state, first-order conditions with respect to d and s are similar to (A2) and (A7). First-order conditions (37) to (39) with specific functions are

$$(A11) \quad \left(\frac{\alpha}{n}\right) T_h(h, m) [(1 - \tau^o) f_l(k, \Omega(h, m), l) - (1 - \pi^o - \xi^o) m] - (1 - \pi - \xi) = 0,$$

$$(A12) \quad T_m(h, m) [(1 - \tau^o) f_l(k, \Omega(h, m), l) - (1 - \pi^o - \xi^o) m] - (1 - \pi^o - \xi^o) T(h, m) = 0,$$

$$(A13) \quad \rho n^{2-\sigma_2} c^{\sigma_1} (1 + \tau^c) - f_l(k, \Omega(h, m), l) [(1 - \tau) v n^2 + \alpha (1 - \tau^o) T(h, m)] + \alpha \left[c \left(\frac{\beta n}{\alpha}\right)^{\frac{1}{\sigma_1}} (1 + \tau^c) + (1 - \xi^o) m T(h, m) - I^o \lambda^o - P - \theta y n + \tau^s (\theta y n - 1) \right] = 0.$$

Given $(\xi, \xi^o, P, \tau^c, \tau^s)$, optimal (ξ^s, τ, τ^o) and $(\pi, \pi^o, I, I^o, \lambda, \lambda^o)$ are as follows:

$$(i) \quad \xi^s = \frac{(n-\alpha)\tau^s}{n},$$

$$(ii) \quad \tau^o = \frac{h\xi - \Lambda_6 \tau^c + \Lambda_7 \xi^o + \frac{k(\alpha-vn)\xi^s}{\alpha v} + \frac{(vn-\alpha(1-vn))P}{vn^2} - \frac{\alpha m(1-vn)f_m(k, \Omega(h, m), l)^2}{v(T_m(h, m)m+T)}}{\Lambda_8},$$

$$(iii) \quad \pi^o = \frac{n f_m(k, \Omega(h, m), l) + \tau^o f_l(k, \Omega(h, m), l) T_m(h, m)}{T_m(h, m)m + T(h, m)} - \xi^o,$$

$$(iv) \quad \tau = \left(\frac{1}{v f_l(k, \Omega(h, m), l)} \right) \left\{ \frac{\alpha [T(h, m)(m \xi^o - f_l(k, \Omega(h, m), l) \tau^o) + P + m n f_m(k, \Omega(h, m), l) \pi^o]}{n^2} + \tau^c \left[\frac{T(h, m) \alpha (m - f_l(k, \Omega(h, m), l))}{n^2} - v f_l(k, \Omega(h, m), l) - k \right] - k \xi^s \right\},$$

$$(v) \quad \pi = f_h(k, \Omega(h, m), l) \left(1 - \frac{T_m(h, m) \alpha m}{T_m(h, m)m + T(h, m)} \right) + \frac{f_l(k, \Omega(h, m), l) T_h(h, m) \alpha T(h, m) \tau^o}{n [T_m(h, m)m + T(h, m)]} + \frac{[n(1 - f_h(k, \Omega(h, m), l)) - T_h(h, m) \alpha (f_l(k, \Omega(h, m), l) - m)] \tau^c}{n} - \xi,$$

$$(vi) \quad I = h\pi,$$

$$(vii) \quad I^o = mT(h, m)\pi^o$$

$$(viii) \quad \lambda = f_h(k, \Omega(h, m), l),$$

$$(ix) \quad \lambda^o = \frac{n f_m(k, \Omega(h, m), l)}{T(h, m)}.$$

Substituting (i) and $s = kn$ into (A7) yields (A3). Substituting (ii), (iii) and (v) into (A11) yields (A4). Substituting (ii), and (iii) into (A12) yields (A5). Substituting conditions (i) to (iv), (vii) and (ix) into (A13) yields (A6). Substituting market clearing conditions, the zero-profit condition in production, the zero-profit condition for health insurance provision, the government budget constraint, (vi) and (vii) into (36) can recover feasibility in (7) at the steady state or optimal c in (A1). Conditions (viii) and (ix) follow (29) and (30). Finally, conditions (i), (ii) and (iv) satisfy the government balanced budget constraint.